Profit Sharing in Partnerships: Complementarity, Productivity, and Commitment*

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April 12, 2024

Abstract

In a partnership game, a principal and an agent negotiate over their profit-sharing rule, after which each individually chooses effort, generating profits. We study the roles of complementarity in efforts, asymmetric productivity, and timing of effort choices in profit-sharing partnerships. When the agent is relatively more productive than the principal, the agent gets lower bargaining power under a stronger degree of complementarity. The surplus of the partnership is always higher in the case of sequential effort choice than in the simultaneous-choice counterpart. We provide implications for allocation of ownership in corporate governance, surplus maximization in partnerships, and optimal hiring.

Keywords: Partnership, Ownership Structure, Profit-Sharing Rule, Negotiation, Complementarity.

JEL Classifications: C72, C78, G32, L14

*We thank Jay Pil Choi, Joel Watson, and participants at the Yonsei micro reading group seminar for valuable comments and suggestions. This work was supported by the Yonsei Economics Research Fund (R. K. Cho Research Cluster Program).

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1 Introduction

Profit-sharing partnerships are common in human-capital-intensive professional services (Levin and Tadelis 2005). For example, a business owner needs a manager to run the business together or hires a management consultant to reorganize their business. Lawyers and attorneys in the same field often work together at the same firm or start their law firm as partners and associates. Firms engage in product or technology partnerships to develop new products or improve existing ones. A principal investigator brings other researchers or institutions to form an R&D consortium for research and commercialization.

When these partnerships are contractual, the two parties—a principal and an agent—can first negotiate over a profit-sharing rule, after which they take productive actions (or exert efforts) that generate the return or profit in their partnership. The return might depend not only on individual-specific contributions of the two parties' efforts but also on a joint contribution of their efforts. The former component is determined by each party's productive ability, captured by heterogeneity in their productivity of efforts; the latter component is governed by the degree of complementarity in their efforts. Further, the return from their teamwork might also depend on whether they exert efforts simultaneously or sequentially.

In this paper, we study how *productivity* of efforts, *complementarity* in efforts, and *commitment* of the principal's effort affect profit sharing in partnerships. A profit-sharing rule dictates an allocation of ownership, characterized by each party's bargaining power or weight in the final negotiation over how to share the return or profit from their partnership. We characterize the combined effects of complementarity and asymmetric productivity on the equilibrium bargaining weights in negotiation, and compare those effects between the two cases in which the timing of effort choices is either simultaneous or sequential. Our analysis provides managerial implications for contracting and governance structure regarding commitment, surplus maximization in partnerships, and employee selection process.

We consider a partnership game with multi-stage negotiations of the following form: In the initial negotiation phase, a principal (who initially has all the bargaining power) and an agent jointly decide their profit-sharing rule—that specifies their bargaining weights in the final negotiation—and make an immediate transfer. In the action phase, the players individually choose their effort levels either simultaneously or sequentially (with the principal choosing first). In the final negotiation phase, the two players jointly decide the terms of compensation, i.e., the sharing of the realized revenue. At this stage, the players have bargaining weights that were agreed upon in the initial negotiation. We use the concept of negotiation equilibrium where joint decisions conform to the standard bargaining solution.

We first present the benchmark case in which the two players have no difference in productivity. This benchmark allows us to focus on the effect of complementarity in efforts on the optimal bargaining weight, and compare those effects between the two games that differ in the timing of effort choices. We find that when the players choose efforts simultaneously in the action phase, the equilibrium entails an equal bargaining weight chosen in the initial negotiation for splitting the return in the final negotiation, regardless of the degree of complementarity; however, when the principal chooses her effort first, then the agent gets more than half the bargaining weight.[1] This is because the marginal contribution of each player to the surplus is symmetric, and so the surplus is maximized when each player has an equal share in the simultaneous-choice case. By contrast, the principal choosing first would be willing to give more share to the agent in order to induce him to exert enough effort that further complements in maximizing the total surplus.

We generalize our analysis to an environment in which the two players have

[1] We use female pronouns for the principal and male pronouns for the agent.

heterogeneous productivity of efforts in generating the return. We show three main results. First, in both games, given any degree of complementarity, the more productive the agent, the higher bargaining weight he receives in equilibrium. This result is intuitive: A more productive partner gets a higher share of the return. Second, in both games, given the agent's relatively higher productivity, the stronger the complementarity, the lower bargaining weight the agent receives. This is because a higher degree of complementarity creates more room for the less productive principal to contribute to the surplus by increasing her effort level. A higher share to the principal, on the one hand, will increase the principal's effort and thus the surplus; on the other hand, it will decrease the agent's effort and thus the surplus. But the principal's marginal contribution to the surplus is big enough so that the former effect offsets the latter.

Third, given some degree of complementarity, the principal receives a higher bargaining weight in the game with sequential effort choice than in the game with simultaneous effort choice *if the agent is sufficiently more productive than the principal*, reversing the benchmark result. The principal who is choosing first is willing to give the agent more share (than when choosing simultaneously) to induce him to exert more effort. But if the agent is much more productive, the effect that the principal has on the surplus by exerting more effort, induced by getting more share herself, is greater when she chooses first than when choosing simultaneously. The latter effect outweighs the former effect.

Our analysis delivers several managerial implications. First, with stronger complementarity, allocating more bargaining power (or share of ownership) to the *less* productive partner—whether it is the principal or the agent—maximizes the surplus and is to the principal's benefit. Second, the principal's commitment to effort matters when the partnership exhibits complementarity in partners' efforts, whereas it does not affect the optimal allocation of power regardless of heterogeneous productivity when there is no complementarity. Third, there are always surplus gains when the players choose efforts sequentially relative to when they choose simultaneously, and such gains are always higher for a higher degree of complementarity or a higher productivity of the agent. This implies that the principal can benefit more from commitment by partnering with the agent of a high level of productivity or complementarity. Lastly, our results have an interesting implication for optimal hiring of an agent in a partnership. The principal shows an increasing marginal rate of substitution between the agent's productivity and complementarity. This implies that in the perspective of the principal selecting her partner among the pool of potential agents who differ in the degree of complementarity and the level of productivity, hiring an "all-rounder" partner may not necessarily be optimal.

Related Literature. Our paper is related to a large body of contract theory literature. The relational contracts literature has provided insights on self-enforcement of agreements in environments with repeated play and limited external enforcement (e.g., Levin 2003; Miller and Watson 2013) and renegotiation in environments with external enforcement of long-term contractual relationships (e.g., Watson, Miller and Olsen 2020). Another line of literature examines the hold-up problem in short-term trading relationships (e.g., Aghion, Dewatripont and Rey 1994; Hart and Moore 1988).[2] Most works in the contract literature have focused on the theoretical properties of contractual relations, providing insights into characterizing optimal contracts in settings of moral hazard or hidden information. Our focus is not on those issues related to enforcement, renegotiation, or hold-up, but on discovering how the technological details of contractual settings influence outcomes

[2] Several papers show that if the contract could specify the ownership and its timing of enforcement, then the contract achieves efficient investments or efforts despite them being unobserved (e.g., Demski and Sappington 1991; Edin and Hermalin 2000; Nöldeke and Schmidt 1998).

in partnerships.[3] Those details include the nature of individual actions (productivity and complementarity) and the timing of actions (simultaneous or sequential).

Our model of teamwork and negotiations is a direct extension of a partnership game with joint decisions presented in Watson (2013, Chap. 21, ex. 10). To analyze such a game, we can combine subgame perfection with the standard bargaining solution, together defining a negotiation equilibrium that identifies behavior at both individual- and joint-decision nodes.[4] While we closely follow the tools of Watson (2013) in solving for an equilibrium, our model is generalized to incorporate various traits of teamwork that are often observed and important in profit-sharing partnerships but were not fully taken into account in Watson's example.

In terms of the theme of our paper—profit-sharing partnerships, the most closely related paper is Levin and Tadelis (2005).[5] They show that profit-sharing partnerships perform better than wage-paying corporations when human capital is crucial in determining product or service quality and when market monitoring of quality is weak. In their model, they assume an equal sharing of profits among partners but note that their results will continue to hold for a broader class of sharing rules that redistribute profits from high-quality partners to marginal partners. In our model, negotiation between partners endogenously determines a profit-sharing rule. The primary interest of our paper is in studying how complementarity and productivity among partners shape the profit-sharing rule, which in turn affects the performance across partnerships with different forms of profit redistribution.

[3] One-period partnership games have been studied by Holmström (1982); repeated partnership games by Radner (1985, 1986) and Radner, Myerson and Maskin (1986). This line of literature focuses on the efficiency property of equilibria in partnerships and not on partnership formation itself, so their games do not have a negotiation stage in which partners decide sharing rules.

[4] A negotiation equilibrium is a special case of contractual equilibrium developed by Miller and Watson (2013).

[5] Related, Liu, Albert Ma and Mak (2018) study a principal contracting with a group of experts for service or production and show that the principal can implement the first best by delegating all decisions to an expert-organization. See also Makris and Siciliani (2013) and Che and Chung (1999) for providing incentives to agents in partnerships.

Lastly, our paper is broadly related to the vast literature on corporate governance system.[6] Zingales (2002) defines a governance system as "the complex set of constraints that shape the ex-post bargaining over the quasi-rents generated in the course of a relationship" (p. 498). In our paper, the returns to a partnership are distributed only when the two parties agree on a profit-sharing rule and an ex-post negotiated compensation or salary. The ex-ante negotiated profit-sharing rule, which reflects the allocation of ownership and organizational structure, affects the process through which quasi-rents are distributed.[7] In the spirit of Zingales's (2002) definition, our model fits in the literature on corporate governance. The literature has been controversial about whether larger equity ownership by managers positively affects corporate performance. Some empirical studies find a significant positive impact of insider ownership on firm performance (e.g., Cole and Mehran 1998; Kaplan 1989). Others argue that the effect of insider ownership is ambiguous due to the issues of endogeneity and dependence on assumptions that researchers have made (e.g., Cheung and Wei 2006; Core, Guay and Larcker 2003; Demsetz and Lehn 1985). Our paper provides theoretical implications on the efficacy of ownership allocation in maximizing profits or total surplus in partnerships.

The rest of this paper is organized as follows. In Section 2, we present a model of partnership game and review the equilibrium concept. In Section 3, we analyze the benchmark case. In Section 4, we present our main results on the effects of complementarity, productivity, and commitment. In Section 5, we discuss managerial implications and related issues. In Section 6, we conclude. All proofs are in Appendix A.

[6]For literature surveys, see Farinha (2003), Shleifer and Vishny (1997), and Zingales (2002).

[7]In our model, the ex-post bargaining over quasi-rents results in the principal extracting all the quasi-rents. See Section 5.1.

2 A Model of Teamwork and Negotiations

2.1 Partnership Game

We consider a modified version of the partnership game with multi-stage negotiations presented in Watson (2013, Chap. 21, ex. 10), from which we adopt his descriptions and notations.

Two players, indexed by $i \in \{1, 2\}$, each contribute effort to an enterprise. Denoting by π_1 and π_2 the two players' bargaining powers (or weights), we assume that initially player 1 has full bargaining power, i.e., $\pi_1 = 1$ and $\pi_2 = 0$. We often refer to player 1 as the principal and player 2 as the agent.

At the initial joint decision stage, players 1 and 2 choose $g \in [0, 1]$, which will be player 1's bargaining weight in the final negotiation. They also agree on a transfer $w \in (-\infty, \infty)$, which is interpreted as an immediate payment from player 2 to player 1 in exchange for player 2's bargaining weight $1 - g$.[8] If they fail to agree, then the game ends, and they each earn zero payoff.

After they agree on (g, w), the players individually choose their effort levels. Let $x \in [0, \infty)$ be player 1's effort and $y \in [0, \infty)$ be player 2's effort. The cost of effort is given by x^2 for player 1 and y^2 for player 2.[9] We consider two scenarios in terms of the timing of effort choices:

SM. Players simultaneously choose their efforts; neither can observe the other's effort when choosing.

SQ. Player 1 first chooses x, and player 2 chooses y after observing x.[10]

[8]The players can agree to allocate some power to player 2 in a way that player 2 buys the shares $1 - g$ of company stock from player 1 at the price of w, becoming a part owner of the company.

[9]The specific functional form for the cost keeps the analysis tractable; we conjecture that the qualitative results of our paper would remain intact for other convex costs.

[10]We discuss briefly in Section 2.2 why we only consider the case of player 1 choosing first.

At the final stage after both players observe x and y, they jointly decide whether and how to split the return of their effort choices. Let $f(x, y)$ denote the revenue from their partnership. In this final negotiation, player 1's bargaining weight is g, and player 2's bargaining weight is $1 - g$. If the players agree on a transfer $t \in (-\infty, \infty)$, then player 1 collects the revenue of $f(x, y)$ and pays t to player 2. The transfer t is interpreted as player 2's salary or dividend. If the two players fail to agree on t, then their partnership dissolves, in which case they receive no revenue but have already incurred their effort costs and made the initial stage transfer of w.

We assume that the revenue increases with each player's effort level, i.e., $f_x \geq 0$ and $f_y \geq 0$; and that the players' efforts are complementary in generating the revenue, i.e., $f_{xy} \geq 0$. For simplicity and tractability of analysis, we consider the following specification of $f(x, y)$:

$$f(x, y) = 2(x + \alpha y + \kappa xy) \tag{1}$$

where the revenue is composed of individual-specific contributions, $x + \alpha y$, and a joint contribution, κxy.[11] Here, $\alpha \in (0, \infty)$ represents the marginal contribution of player 2's effort to the individual-specific component of the revenue relative to player 1's effort. With minor abuse of terminology, we will interpret α as player 2's relative marginal *productivity*.[12] A higher α represents a greater productive ability of player 2 in teamwork, and $\alpha > 1$ characterizes a case in which player 2 is relatively more productive than player 1. The coefficient $\kappa \in [0, 1]$ captures

[11]We scale $(x + \alpha y + \kappa xy)$ by two for analytical simplicity; any monotonic increasing transformation does not affect the qualitative results of our paper. The multiplicative form of the joint contribution κxy is convenient to study.

[12]In the usual sense, the marginal productivity of player 2's effort to the revenue is defined as $f_y(x, y) = \alpha + \kappa x$; however, for our purpose we define it as the marginal productivity that pertains only to the individual-specific component of the revenue. In line with this definition, we assume that the players' asymmetric marginal productivity does not affect the joint contribution component.

the degree of *complementarity* in players' efforts.[13] The case of $\kappa = 0$ represents no complementarity, and the higher κ, the stronger the complementarity in players' efforts.

We will henceforth refer to the partnership game with simultaneous effort choices at the individual decision stage as the **SM** game, and to that with sequential effort choices as the **SQ** game. The game tree is illustrated in Figure 1, adapted from Watson (2013, Chap. 21), with the payoffs indicated at the terminal nodes. Surrounded solid circles represent joint decision nodes, and solid circles represent individual decision nodes. A dashed line around player 2's decision node indicates that player 2 makes his choice without observing player 1's choice, corresponding to the SM game.

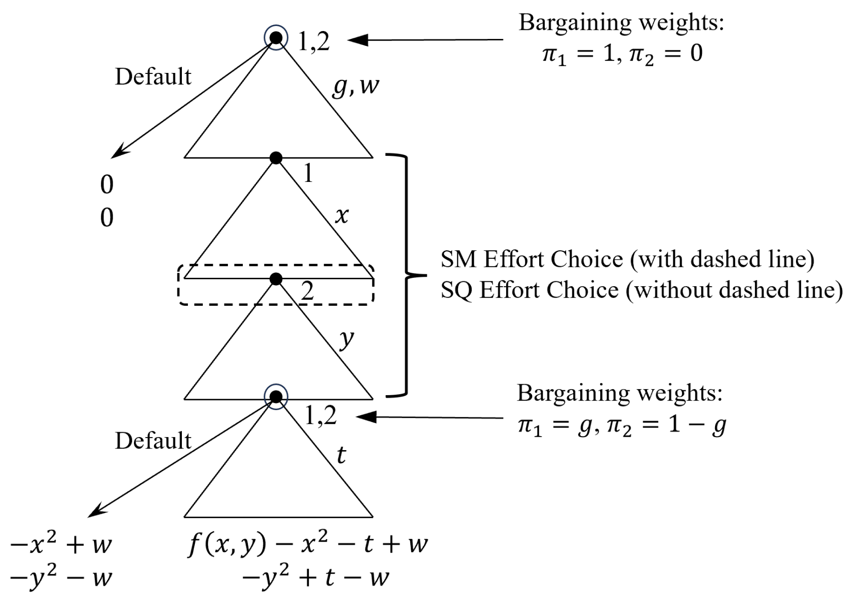


Figure 1: Game tree for the SM and SQ games

[13]The restriction $\kappa \leq 1$ suffices for the uniqueness of equilibrium and an interior solution for effort level. The necessary and sufficient condition for a unique interior solution is $1 > \kappa^2(g(\alpha, \kappa)(1 - g(\alpha, \kappa))$ in the SM game and $1 > 2\kappa^2(g(\alpha, \kappa)(1 - g(\alpha, \kappa))$ in the SQ game, where $g(\alpha, \kappa)$ is endogenously determined in equilibrium.

2.2 Discussion of the Model

We generalize the model of Watson (2013, Chap. 21, ex. 10) in the following three ways. First, we introduce a *productivity* parameter $\alpha \in (0, \infty)$ in the revenue function, allowing for heterogeneity in the players' productivity of efforts. The symmetric case of $\alpha = 1$, in which the two players contribute equally to the teamwork, is a useful benchmark we will examine in Section 3. Second, we introduce a *complementarity* parameter $\kappa \in [0, 1]$. The revenue function in Watson's model is $f(x, y) = x + 2y$, which is a special case of our revenue function with $\alpha = 2$ and $\kappa = 0$. Such generalizations allow us to study the effects of asymmetric productivity and strategic complementarity in efforts on the equilibrium.

Third, we consider the issue of *commitment* by analyzing both the SM and SQ games, while Watson's model only considers the SM game. The SQ game captures the situation in which player 1 commits to her effort before player 2 chooses his effort. The comparison between the SM game and the SQ game provides managerial implications regarding the value of commitment in negotiations over "ex-post" bargaining power (or share). We do not consider the case in which player 2 chooses his effort first, after which player 1 chooses hers. In the literature on the principal-agent problem, a principal refers to the individual who controls an asset or designs and offers a contract, and an agent refers to the one who is "hired" by the principal and takes appropriate actions. We are interested in situations where the principal (player 1), who initially has all the bargaining power, forms a profit-sharing partnership with the agent (player 2). In this sense, it seems natural to study the effect of the principal committing to her action first. Nonetheless, we can derive the equilibrium of the SQ game in which player 2 chooses his effort first, which becomes a mirror image of our results; but this analysis does not add more insights.[14]

[14]The detailed results are available from the authors upon request.

Much literature on human resource management, personnel psychology, and organizational behavior has studied how shared leadership and ownership affect team performance and team members' behaviors. Salas et al. (2015) distinguish taskwork from teamwork: Taskwork involves specific tasks to achieve team goals, whereas teamwork focuses on the shared behaviors, attitudes, and cognitions that are necessary for teams to accomplish the given tasks. According to this distinction, our parameters α and κ can be related to taskwork ability and teamwork, respectively. However, in our paper, teamwork refers to a partnership as a whole in which the two players work together to generate some joint return. So we view both α and κ as the elements of teamwork attributing to a successful partnership.

2.3 Negotiation Equilibrium with Standard Bargaining Solution

The solution concept we use is a *negotiation equilibrium* (Watson 2013), which is a subgame-perfect Nash equilibrium in a game with joint decisions. Each player maximizes her payoff when making effort choices at individual decision nodes, consistent with sequential rationality; and the outcomes of joint decisions (at the initial and final stages of our game) are consistent with the standard bargaining solution for given bargaining weights. Before characterizing the equilibrium of our game, we briefly review the concept of a standard bargaining solution.

Two players, $i = 1, 2$, are participating in bargaining. A bargaining problem is expressed as a *bargaining set* $V = (\mathcal{U}, d)$. In a pair $u = (u_1, u_2) \in \mathcal{U}$, u_1 is player 1's payoff and u_2 is player 2's payoff. The d is a pair (d_1, d_2) referred to as a *default outcome*, which describes each player's payoff with no agreement. Let τ denote a monetary transfer from player 2 to player 1. A negative value of τ indicates a transfer from player 1 to player 2. Let z represent negotiated items, such as whether to form a partnership, profit-sharing rules, salaries, wage rates, compensation schemes, etc. Assuming the additive separability of utility for money, player 1's payoff is $u_1(z, \tau) = v_1(z) + \tau$ and player 2's payoff is $u_2(z, \tau) = v_2(z) - \tau$ for some functions v_1 and v_2. A *joint value* is defined as the sum of two players' payoffs: $u_1(z, \tau) + u_2(z, \tau) = v_1(z) + v_2(z)$.

In settings with transferable utility, an outcome is efficient if and only if it maximizes the players' joint value. Let v^* denote the maximized joint value, which can be calculated by determining the value of z that maximizes $v_1(z) + v_2(z)$. Because each player can unilaterally incur the default outcome, no rational player would accept an agreement that gives her less than her default payoff. Hence, the players negotiate over the *surplus* $v^* - d_1 - d_2$. Bargaining weights summarize bargaining powers in a negotiation. We denote player i's bargaining weight by $\pi_i \geq 0$, where $\pi_1 + \pi_2 = 1$, interpreted as the proportion of the surplus that player i obtains.

The *standard bargaining solution* is a mathematical representation of efficiency and proportional division, in which each player is assumed to obtain her default payoff plus her share of the surplus (Watson 2013). That is, the two players reach an agreement, obtaining their payoffs of $u_1^* = d_1 + \pi_1(v^* - d_1 - d_2)$ and $u_2^* = d_2 + \pi_2(v^* - d_1 - d_2)$. The transfer τ achieves the required split of the surplus, thus satisfying $u_1^* = v_1(z^*) + \tau$ and $u_2^* = v_2(z^*) - \tau$. We use this concept of standard bargaining solution to identify optimal behavior at joint decision nodes. In particular, in constructing the negotiation equilibrium of our game, the specifications of (g, w) at the initial joint decision node and of t at the final joint decision node are pinned down by the standard bargaining solution.

3 Symmetric Productivity Benchmark

In this section, we consider the benchmark case of symmetric productivity with $\alpha = 1$ in both the SM and SQ games. We characterize equilibrium using backward induction and the standard bargaining solution. We then examine the effect of different degrees of complementarity $\kappa \in [0,1]$ on the optimal bargaining weight and compare those effects between the two games.

3.1 Simultaneous Effort Choice

We first examine the SM game. Consider the joint decision node in the players' final negotiation over t given (g, w, x, y). At this node, the joint value is $u_1 + u_2 = (f(x,y) - x^2 - t + w) + (-y^2 + t - w)$ and the default payoffs are $d_1 = -x^2 + w$ and $d_2 = -y^2 - w$. Hence, the surplus of an agreement at the final node is computed as $u_1 + u_2 - d_1 - d_2 = f(x,y)$. According to the standard bargaining solution, the players (with their bargaining weights $\pi_1 = g$ and $\pi_2 = 1 - g$) reach an agreement in which they receive $u_1^* = (-x^2 + w) + gf(x,y)$ and $u_2^* = (-y^2 - w) + (1-g)f(x,y)$. To achieve such a split, player 2's payoff must satisfy $u_2^* = (-y^2 - w) + t$, so we obtain $t^* = (1-g)f(x,y)$. The superscript asterisks (*), henceforth, refer to the optimal solutions for the SM game.

Now consider the subgame where the players individually and simultaneously choose their effort levels. Given g and w, players 1 and 2 each choose x and y, respectively, to maximize their payoffs, expecting the continuation payoffs with $t^* = (1-g)f(x,y)$:

$$\max_{x \in \mathbb{R}^+} \; -x^2 + w + gf(x,y); \quad \text{and} \quad \max_{y \in \mathbb{R}^+} \; -y^2 - w + (1-g)f(x,y). \tag{2}$$

The first order conditions with respect to x and y, respectively, are $gf_x = 2x$ and $(1-g)f_y = 2y$, where $f_x = 2(1+\kappa y)$ and $f_y = 2(1+\kappa x)$. Thus we obtain the solutions for the optimal effort levels, denoted by $x^*(g;\kappa)$ and $y^*(g;\kappa)$, for any given $\kappa \in [0,1]$, as follows:

$$x^*(g;\kappa) = \frac{g + \kappa g(1-g)}{1 - \kappa^2 g(1-g)} \quad \text{and} \quad y^*(g;\kappa) = \frac{(1-g) + \kappa g(1-g)}{1 - \kappa^2 g(1-g)}. \tag{3}$$

Lastly, consider the initial joint decision node where the two players jointly choose g and w to maximize the surplus that they get under an agreement of those values in equilibrium. At the initial node, the joint value is $\big(-x^*(g;\kappa)^2 + w + gf(x^*(g;\kappa), y^*(g;\kappa))\big) + \big(-y^*(g;\kappa)^2 - w + (1-g)f(x^*(g;\kappa), y^*(g;\kappa))\big)$, and the default payoffs are zero for both players. Hence, the surplus of an agreement at the initial node is computed as $f(x^*(g;\kappa), y^*(g;\kappa)) - x^*(g;\kappa)^2 - y^*(g;\kappa)^2$. Then the optimal g, denoted by $g^*(\kappa)$, solves the following:

$$g^*(\kappa) = \underset{g\in[0,1]}{\arg\max}\, S^*(g) := 2\big(x^*(g)+y^*(g)+\kappa x^*(g)y^*(g)\big) - x^*(g)^2 - y^*(g)^2, \tag{4}$$

where the parameter κ is suppressed on the right-hand side for simplicity. The optimal w^* can be found by setting each player's equilibrium payoff equal to her default payoff (which is zero when the initial negotiation fails) plus her share of the maximized surplus given weights $\pi_1 = 1$ and $\pi_2 = 0$, according to the standard bargaining solution.

The following lemma characterizes the equilibrium $(g^*, w^*, x^*, y^*, t^*)$.

Lemma 1. *In the SM game with $\alpha = 1$, for any given $\kappa \in [0,1]$, the unique negotiation equilibrium is characterized as follows:*

$$g^* = \frac{1}{2},\ w^*(\kappa) = \frac{3-\kappa}{(2-\kappa)^2},\ x^*(\kappa) = \frac{1}{2-\kappa},\ y^*(\kappa) = \frac{1}{2-\kappa},\ t^*(\kappa) = \frac{4-\kappa}{(2-\kappa)^2}.$$

Lemma 1 shows that in the SM game with symmetric productivity, the players jointly agree on an equal allocation of bargaining power in equilibrium regardless of the degree of complementarity. This result is intuitive because the surplus function is symmetric in the two players' efforts, as shown in (4). Symmetry implies that the surplus is maximized when the two players choose the same level of effort, i.e., $x^*(g;\kappa) = y^*(g;\kappa)$, which is induced only by the equal bargaining weight, $g^* = 1/2$, regardless of κ.

Lemma 1 also delivers intuitive comparative statics with respect to the degree of complementarity κ. The optimal "price" $w^*(\kappa)$ that player 2 pays for an equal bargaining power, the optimal efforts $(x^*(\kappa), y^*(\kappa))$, and the optimal "dividend" $t^*(\kappa)$ are all increasing in κ. These are because the realized surplus is monotonically increasing in κ.

3.2 Sequential Effort Choice

We now examine the SQ game. At the joint decision node in the players' final negotiation over t given (g, w, x, y), the standard bargaining solution implies that the optimal transfer, denoted by $\hat{t}$, must satisfy $\hat{t} = (1-g)f(x,y)$ as in the SM game. The "hats" $(\hat{\cdot})$ distinguish the optimal solutions for the SQ game from the SM counterparts. Players 1 and 2 respectively obtain $-x^2 + w + gf(x,y)$ and $-y^2 - w + (1-g)f(x,y)$ under an agreement.

In the subgame where the players choose their effort levels sequentially, player 1 first chooses x given (g, w) to solve

$$\max_{x \in \mathbb{R}^+} \big(-x^2 + w + gf(x,y)\big) \quad \text{subject to} \quad (1-g)f_y - 2y = 0,$$

where the constraint is player 2's first order condition that defines his best response. Recall that $f(x,y) = 2(x+y+\kappa xy)$ and $f_y = 2(1+\kappa x)$. Then player 1's maximization problem reduces to: $\max_{x\in\mathbb{R}^+}\ (-x^2 + w) + 2g\left(x + (1-g)(1+\kappa x)^2\right)$. The first order condition with respect to x yields $-2x + 2g(1 + 2\kappa(1-g)(1+\kappa x)) = 0$.

Solving for x, we obtain the optimal effort by player 1, denoted by $\hat{x}(g;\kappa)$; then plugging $x = \hat{x}(g;\kappa)$ into player 2's first order condition, $y = (1-g)(1+\kappa x)$, we obtain the optimal effort by player 2, denoted by $\hat{y}(g;\kappa)$, for any given $\kappa \in [0,1]$:

$$\hat{x}(g;\kappa) = \frac{g + 2\kappa g(1-g)}{1 - 2\kappa^2 g(1-g)} \quad \text{and} \quad \hat{y}(g;\kappa) = \frac{(1-g) + \kappa g(1-g)}{1 - 2\kappa^2 g(1-g)}. \tag{5}$$

Lastly, consider the initial joint decision node where the two players jointly choose g and w to maximize the surplus, which is computed in the same way as in the SM game: $\widehat{S}(g) \equiv 2\big(\hat{x}(g) + \hat{y}(g) + \kappa\hat{x}(g)\hat{y}(g)\big) - \hat{x}(g)^2 - \hat{y}(g)^2$, where κ is suppressed. The optimal g maximizes $\widehat{S}(g)$, and the optimal $\hat{w}$ is found according to the standard bargaining solution.

The following lemma describes the unique negotiation equilibrium $(\hat{g}, \hat{w}, \hat{x}, \hat{y}, \hat{t})$ in the SQ game. Unlike in the SM game, the explicit formulae that characterize the equilibrium are cumbersome to state here, so we focus on the underlying intuition.

Lemma 2. *In the SQ game with $\alpha = 1$, for any given $\kappa \in [0,1]$,*

(i) The equilibrium bargaining weight for player 1, denoted by $\hat{g}(\kappa)$, is characterized by the quartic equation (A.3) in Appendix A.

(ii) There is a unique real solution to (A.3) such that $\hat{g}(\kappa) \le 1/2$ for all $\kappa \in [0,1]$ where the equality holds iff $\kappa = 0$.

(iii) The equilibrium efforts $(\hat{x}(\kappa), \hat{y}(\kappa))$ are obtained by plugging in $g = \hat{g}(\kappa)$ in (5). The equilibrium transfers are characterized by $\hat{w}(\kappa) = \hat{t}(\kappa) - \hat{y}(\kappa)^2$ and $\hat{t}(\kappa) = (1 - \hat{g}(\kappa))f(\hat{x}(\kappa), \hat{y}(\kappa))$.

We elaborate on the equilibrium bargaining weight of the SQ game in comparison to that of the SM game in the next subsection. Note that it can be shown that the optimal efforts $(\hat{x}(\kappa), \hat{y}(\kappa))$ and transfers $(\hat{w}(\kappa), \hat{t}(\kappa))$ are all increasing in κ, as in the SM game. An interesting observation is the comparison of the equilibrium effort choices between the SM game and the SQ game. Figure 2 illustrates player 1's optimal effort choices with respect to $\kappa \in [0, 1]$ in the two games.

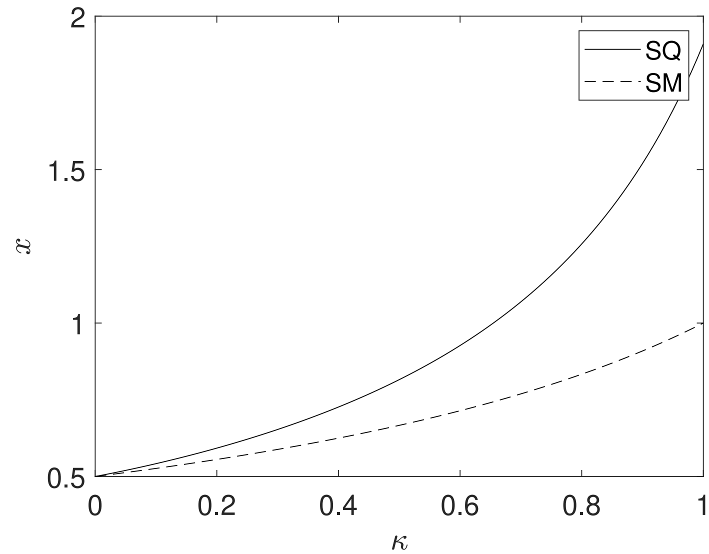


Figure 2: Player 1's efforts x^* (SM) and $\hat{x}$ (SQ)

The figure shows that $\hat{x}(\kappa) \geq x^*(\kappa)$ for all $\kappa \in [0, 1]$, and that the difference between the two variables increases in κ. In providing the intuitions, we find it useful to decompose the two channels through which κ affects the equilibrium x. First, in both the SM and SQ games, κ directly increases the marginal benefit to player 1 of raising x, which we refer to as a *direct* effect. Second, in the SQ game, κ increases the marginal benefit to player 1 of raising x through y. This channel is called a *strategic* effect. To see this formally, consider player 1's payoff change with respect to x in the SQ game:

$$\frac{du_1}{dx} = -2x + g \underbrace{f_x}_{direct} + g \underbrace{f_y(dy/dx)}_{strategic}.$$

The $f_x = 2(1 + \kappa y)$ in the second term increases in κ, capturing the direct effect in both games. In the SQ game, player 1 has an additional incentive to raise x for a higher marginal revenue through y because player 1 takes into account that player 2's effort choice is determined by $(1-g)f_y - 2y = 0$, such strategic effect of which is captured by $f_y(dy/dx) = 2(1+\kappa x) \times \frac{(1-g)f_{yx}}{2-(1-g)f_{yy}} = 2(1+\kappa x)(1-g)\kappa$ where $f_{yx} = 2\kappa$ and $f_{yy} = 0$. If $\kappa = 0$, then $f_y \frac{dy}{dx} = 0$, so the equilibrium of the SQ game is equivalent to that of the SM game. If $\kappa > 0$, then $f_y \frac{dy}{dx} > 0$ increases in κ given $g \neq 1$. Thus player 1 has a greater incentive to raise x in the SQ game than in the SM game, which intensifies as κ increases. The dashed curve for the SM game in Figure 2 only includes the direct effect, whereas the solid curve for the SQ game captures both the direct and strategic effects.

3.3 Equilibrium Bargaining Weights

Comparing Lemma 1 and Lemma 2(ii), we establish two immediate observations: (1) when there is no complementarity ($\kappa = 0$), $\hat{g}(\kappa) = g^* = 1/2$; and (2) when there is complementarity ($\kappa > 0$), $\hat{g}(\kappa) < g^* = 1/2$. Figure 3 illustrates these observations.

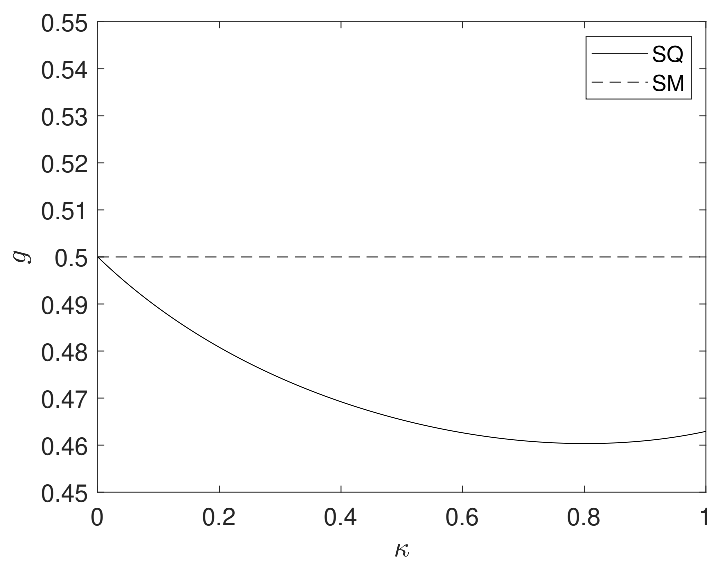


Figure 3: The equilibrium bargaining weights g^* (SM) and $\hat{g}$ (SQ)

The first observation regards the simplest case where the players have equal productivity ($\alpha = 1$) with no complementarity in efforts ($\kappa = 0$). In such a case, the players agree on allocating equal bargaining power for the final negotiation regardless of whether they choose efforts simultaneously or sequentially; this is because the players equally contribute to the total surplus. An implication is that the timing of effort choices does not matter for the optimal profit-sharing rule when there is no complementarity in efforts. This conclusion remains valid when asymmetric productivity ($\alpha \neq 1$) is introduced in the next section (see Remark 2 (i)).[15]

The second observation states that when the players have symmetric productivity, player 1 initially endowed with all the bargaining power allocates equal bargaining power to player 2 regardless of the degree of complementarity in efforts when they choose efforts simultaneously; however, when player 1 chooses effort first, she allows more than half the power to player 2 in the presence of complementarity. This result is intuitive. Given that player 1 has the same productivity in generating the surplus as player 2 does, player 1 who chooses first would be willing to give more share to player 2 in order to induce him to choose sequentially "enough" effort that further complements in maximizing the total surplus. However, we will show later that this conclusion that $\hat{g}(\kappa) < g^*$ is reversed for some high enough values of $\kappa > 0$ and $\alpha > 1$ when allowing for asymmetric productivity (see Remark 2 (ii)).

For the remainder of this section, we explain the intuition behind the second observation in detail by examining how the surplus changes with respect to the bargaining weight. At the initial negotiation stage, the optimal g is chosen to maximize the surplus, denoted by $S(x(g), y(g))$. The first order condition is

$$\frac{dS(x(g), y(g))}{dg} = S_x \frac{dx(g)}{dg} + S_y \frac{dy(g)}{dg} = 0,$$

[15] When $\alpha \neq 1$, the equilibrium bargaining weight in the SQ game is no longer equal to half and varies depending on α given κ. If $\kappa = 0$, then $\hat{g}$ and g^* coincide, both of which depend on α.

which implies that the optimal $\hat{g}$ must be less than half if $dS/dg < 0$ evaluated at $g = 1/2$ in the SQ game. Note that in the SM game, both players choose the same level of effort, $x^* = y^*$, thus $S_x = S_y$ at the optimum; and $dx/dg > 0$ and $dy/dg < 0$ exactly balance out at $g = 1/2$ regardless of κ. Recall that for the given optimal g, both players exert more effort in the SQ game than in the SM game due to the additional strategic effect when $\kappa > 0$. Further, player 1's optimal effort $\hat{x}$ is higher than player 2's optimal effort $\hat{y}$. This implies that $S_x < S_y$ (evaluated at the optimum); that is, the marginal contribution to the surplus from a small change in x is smaller than that from a small change in y. Because $dx/dg > 0$ and $dy/dg < 0$, negotiation over g must end up with a higher share for player 2 to induce more of his effort y.

Let us consider a numerical example where $\kappa = 1$. In the SM game, we have $g^* = 1/2$, $x^* = y^* = 1$, and $S^* = 4$. Suppose that at the initial negotiation in the SQ game, the two players agreed on $\tilde{g} = 1/2$, in which case the subsequent effort choices would be $\hat{x}(\tilde{g}) = 2$ and $\hat{y}(\tilde{g}) = 1.5$. The marginal contributions to the surplus evaluated at these effort levels are $S_x = 1$ and $S_y = 3$; and $d\hat{x}/dg = 2$ and $d\hat{y}/dg = -2$ evaluated at $\tilde{g} = 1/2$. Then we have $\frac{dS}{dg}\Big|_{g=1/2} = (1)(2) + (3)(-2) < 0$, which implies that $\tilde{g} = 1/2$ was not optimal and that optimality calls for a downward adjustment in the bargaining weight. In fact, given the weight $\tilde{g} = 1/2$, player 2's incentive to exert efforts is relatively weak after observing player 1's effort choice. So the players must agree on a lower-than-half g, under which player 2 will choose a higher y than $\hat{y}(\tilde{g}) = 1.5$, lowering the absolute value of $d\hat{y}/dg$; and player 1 will choose a lower x than $\hat{x}(\tilde{g}) = 2$, raising the value of $d\hat{x}/dg$. In the SQ game, the surplus is maximized at $\hat{g} \approx 0.463$, in which case we have $\hat{x} \approx 1.91$, $\hat{y} \approx 1.56$, and $\hat{S} \approx 6.82$. The marginal contributions to the surplus evaluated at the optimum are $S_x = 1.3$ and $S_y = 2.7$; and $d\hat{x}/dg \approx 2.85$

and $d\hat{y}/dg \approx -1.38$. Thus we have $\frac{dS}{dg}\Big|_{g\approx 0.463} \approx (1.3)(2.85) + (2.7)(-1.38) \approx 0$. Note that the marginal contributions to the surplus are $S_x = S_y = 2$ at the optimum in the SM game. This implies that in the SQ game, compared to the SM game, the impact of changing g on the surplus is more than twice as great through dy/dg than through dx/dg.

Another observation in Figure 3 is that $\hat{g}$ in the SQ game is not necessarily decreasing in κ. Using the implicit function theorem on the first order condition for the optimal g, we can derive $d\hat{g}(\kappa)/d\kappa$ of which the sign is not necessarily negative. In the SQ game, $\hat{g}$ and κ interplay through the two players' effort choices and the surplus at the optimum in a subtle way. The key force at work here is the strategic effect that exists only in the SQ game, explained in the previous subsection.

We summarize our main findings of this section as follows:

Remark 1. Consider our benchmark model with $\alpha = 1$ given $\kappa \in [0, 1]$.

(i) With no complementarity in efforts ($\kappa = 0$), the SQ game is equivalent to the SM game; in particular, $g^* = \hat{g}(0) = 1/2$.

(ii) The equilibrium bargaining weight for player 1 (the principal) is lower in the SQ game than in the SM game; that is, $\hat{g}(\kappa) \leq g^* = 1/2$ where the equality holds iff $\kappa = 0$.

(iii) The equilibrium bargaining weight for player 1 in the SQ game is not monotonically decreasing in κ.

Based on Remark 1, we offer two managerial implications for settings with symmetric productivity. Parts (i) and (ii) together imply that commitment does not matter when there is no complementarity in players' actions.[16] In other words, the ability of the principal's commitment to her action before the agent (player 2) chooses his action becomes a more important managerial concern when the partners

[16]This conclusion is also valid for settings with asymmetric productivity (see Remark 2).

expect complementarity in their actions. Further, part (iii) suggests that greater complementarity does not necessarily mean that giving more shares to the agent will benefit the principal. While assigning more bargaining power to the agent is one way to boost his effort, the marginal contributions to the surplus change non-monotonically with the (adjusted) level of efforts and the degree of complementarity together. Thus allocating more shares to the agent is not always optimal when complementarity is greater.[17]

4 Asymmetric Productivity and Complementarity

In this section, we consider the general case of our model with $\alpha > 0$ and $\kappa \in [0, 1]$. We first solve for the equilibrium bargaining weight in both the SM and SQ games. We then study the effects of complementarity and asymmetric productivity on the optimal bargaining weight, and compare those effects between the two games.

4.1 Equilibrium Bargaining Weights

Recall that the revenue from partnership, or the return of effort choices, is represented by $f(x, y) = 2(x + \alpha y + \kappa xy)$ where $\alpha > 0$ and $\kappa \in [0, 1]$. The explicit formula for a closed-form solution of equilibrium bargaining weight is quite cumbersome to state here due to the tedious algebra involved; instead, we provide some general properties of the solutions.

Proposition 1. *In the SM game with* $\alpha > 0$ *and* $\kappa \in [0, 1]$*,*

(i) The equilibrium bargaining weight for player 1, denoted by $g^*(\alpha, \kappa)$*, is characterized by the quadratic equation* (A.10) *in Appendix A.*

[17] When we introduce asymmetric productivity, the degree of complementary affects the optimal bargaining weight differently depending on the size of α, which we will examine in the next section.

(ii) There is a unique real solution $g^*(\alpha, \kappa) \in (0, 1)$ *to* (A.10) *for all* $\kappa \in [0, 1]$ *and* $\alpha > 0$.

(iii) For all $\kappa \in [0, 1]$*,* $g^*(\alpha, \kappa) \leq 1/2$ *if* $\alpha \geq 1$ *(where the equality holds iff* $\alpha = 1$*) and* $g^*(\alpha, \kappa) > 1/2$ *if* $\alpha \in (0, 1)$.

There is no dynamic interplay between effort choices under the SM game. So, the key factor determining the joint decision on the bargaining weight (or share) is how the two players marginally contribute to the total surplus. When $0 < \alpha < 1$, player 1's marginal contribution is higher than player 2's. Thus the share allocated to player 1 should be greater than that to player 2. Conversely, when $\alpha > 1$, player 2 needs to be incentivized for his effort through a more-than-half share.

Proposition 2. *In the SQ game with* $\alpha > 0$ *and* $\kappa \in [0, 1]$,

(i) The equilibrium bargaining weight for player 1, denoted by $\hat{g}(\alpha, \kappa)$*, is characterized by the quartic equation* (A.13) *in Appendix A.*

(ii) There is a unique real solution $\hat{g}(\alpha, \kappa) \in (0, 1)$ *to* (A.13) *for all* $\kappa \in [0, 1]$ *and* $\alpha > 0$.

(iii) For given $\alpha > 0$ *and* $\kappa \in [0, 1]$*,* $\hat{g}(\alpha, \kappa) \leq 1/2$ *if and only if (a)* $\alpha \geq 1$ *or (b)* $1/2 \leq \alpha < 1$ *and* $(1/2)(\sqrt{8 - 7\alpha^2} - \alpha) \leq \kappa \leq 1$*, where* $\hat{g}(\alpha, \kappa) = 1/2$ *if and only if* $1/2 \leq \alpha \leq 1$ *and* $\kappa = (1/2)(\sqrt{8 - 7\alpha^2} - \alpha)$.

Unlike in the SM game, whether player 2 gets more than half share in equilibrium depends on both player 2's relatively higher productivity and the degree of complementarity. In particular, even when player 2 is relatively less productive than player 1 ($1/2 \leq \alpha < 1$), player 2 can get his share greater than half for some sufficiently high degree of complementarity. The threshold for κ increases as α decreases to half.

4.2 Comparative Statics of the Equilibrium Bargaining Weights

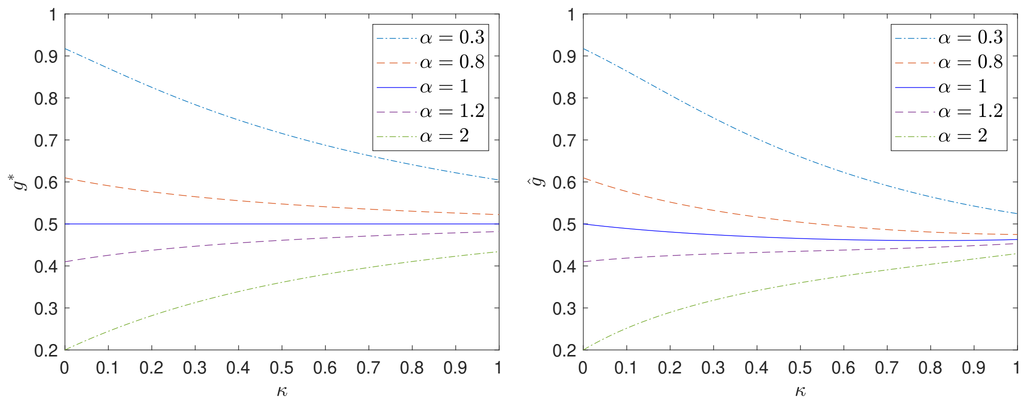
We first characterize the comparative statics of the equilibrium bargaining weight with respect to asymmetric productivity and the degree of complementarity in the SM game.

Proposition 3. *Consider the SM game with $\alpha > 0$ and $\kappa \in [0, 1]$.*

(i) The equilibrium bargaining weight $g^*(\alpha, \kappa)$ strictly decreases in α for any given $\kappa \in [0, 1]$.

(ii) The equilibrium bargaining weight $g^*(\alpha, \kappa)$ strictly decreases in κ if $\alpha \in (0, 1)$, stays the same in κ if $\alpha = 1$, and strictly increases in κ if $\alpha > 1$.

Figure 4(a) illustrates Proposition 3. Part (i) is intuitive: Given the degree of complementarity, the more productive player 2 is, the more bargaining weight he receives in equilibrium. Part (ii) states that given player 2's relatively higher (resp. lower) productivity, the stronger the complementarity, the lower (resp. higher) bargaining weight player 2 gets. This result may seem counter-intuitive because as the efforts complement each other, the players may want to give a higher share to a relatively more productive player. To understand why this is not the case, consider the SM game where player 2 is more productive than player 1 ($\alpha > 1$). The surplus of an agreement at the initial negotiation is $S = 2(x + \alpha y + \kappa xy) - x^2 - y^2$ for x and y chosen optimally at the subsequent nodes. A higher degree of complementarity (κ increases) then means that there is bigger room for (the less productive) player 1 to contribute to the surplus by increasing her effort level through the joint contribution term κxy. To motivate more efforts by player 1, the optimal g must be higher, i.e., the relatively less productive player 1 gets more shares. Because $dx^*/dg > 0$ and $dy^*/dg < 0$, a higher g will decrease player 2's efforts; but player 1's marginal contribution to the surplus by a small increase in x is big enough so that the increase in the surplus by a higher x will offset the decrease in the surplus by a lower y.



(a) The SM game

(b) The SQ game

Figure 4: The equilibrium bargaining weights g^* and $\hat{g}$ with respect to α and κ

Let us explain the logic using numerical examples. Recall that the first order condition at the initial negotiation stage is given by $dS/dg = S_x(dx/dg) + S_y(dy/dg) = 0$. In the SM game with $\alpha = 2$ and $\kappa = 0$, the equilibrium consists of $g_0^* = 0.2$, $x^*(g_0^*) = 0.2$, and $y^*(g_0^*) = 1.6$. The first order condition evaluated at the equilibrium is:

$$\left.\frac{dS}{dg}\right|_{g=g_0^*} = (1.6)(1) + (0.8)(-2) = 0.$$

Now let us consider that the SM game with the same $\alpha = 2$ but $\kappa = 1$. Suppose that the players agreed on the bargaining weight $\tilde{g} = 0.2$, given which the players will choose $x(\tilde{g}) \approx 0.619$ and $y(\tilde{g}) \approx 2.095$. Then the change in the surplus with respect to g evaluated at these effort levels is:

$$\left.\frac{dS}{dg}\right|_{g=\tilde{g}} \approx (4.952)(3.061) + (1.048)(-0.170) \approx 14.982 > 0,$$

which implies that $\tilde{g} = 0.2$ was not optimal and calls for an upward adjustment in the bargaining weight. In fact, given $\alpha = 2$ and $\kappa = 1$, the equilibrium consists of $g_1^* \approx 0.434$, $x^*(g_1^*) \approx 1.227$, and $y^*(g_1^*) \approx 1.826$, in which case the first order condition evaluated at the equilibrium is:

$$\left.\frac{dS}{dg}\right|_{g=g_1^*} \approx (3.2)(1.89) + (2.8)(-2.16) \approx 0.$$

Given $\kappa = 1$, the joint contribution terms are calculated as $x(\tilde{g})y(\tilde{g}) \approx 1.297$ and $x^*(g_1^*)y^*(g_1^*) \approx 2.24$. The equilibrium g and subsequently the equilibrium efforts x and y are chosen so that the joint contribution term κxy is maximized.[18]

The next proposition describes the comparative statics of the equilibrium bargaining weight with respect to asymmetric productivity and the degree of complementarity in the SQ game. We state the proposition without analytical proof, but a graphical illustration is given in Appendix A.

Proposition 4. *Consider the SQ game with $\alpha > 0$ and $\kappa \in [0, 1]$.*

(i) The equilibrium bargaining weight $\hat{g}(\alpha, \kappa)$ strictly decreases in α for any given $\kappa \in [0, 1]$.

(ii) The equilibrium bargaining weight $\hat{g}(\alpha, \kappa)$ strictly decreases in κ if $\alpha \in (0, \underline{\alpha})$, is non-monotonic in κ if $\alpha \in [\underline{\alpha}, \bar{\alpha})$, and strictly increases in κ if $\alpha \geq \bar{\alpha}$, where $0.8 < \underline{\alpha} < 1$ and $1 < \bar{\alpha} < 1.2$.[19]

Figure 4(b) illustrates Proposition 4. The comparative statics in the SQ game is algebraically more complex to prove but is qualitatively similar to that in the SM game, as shown in the two panels of Figure 4. For part (i), the intuition is the same as in Proposition 3(i): for any given κ, the share $\hat{g}(\alpha, \kappa)$ that player 1 receives monotonically decreases as player 2 becomes relatively more productive (higher α).

[18] As an illustration, suppose that given $\alpha = 2$ and $\kappa = 1$, the players (suboptimally) agreed on $g' = 0.5$. Then $x(g') \approx 1.33$, $y(g') \approx 1.67$ so that $dS/dg \approx (2.67)(1.33)+(3.33)(-2.67) \approx -5.33$ (evaluated at $x(g')$ and $y(g')$), and the joint contribution term is $x(g')y(g') \approx 2.22 < 2.24 \approx x^*(g_1^*)y^*(g_1^*)$.

[19] Due to analytical complexity we cannot compute the exact values of the bounds, but they are calculated using MATLAB to be approximately $\underline{\alpha} \approx 0.845$ and $\bar{\alpha} \approx 1.117$.

For part (ii), the comparative statics pattern and the underlying intuition are identical to those in the SM game, except for near $\alpha = 1$. That is, when either player 1 or 2 is sufficiently more productive than the other, i.e., $\alpha < \underline{\alpha} < 1$ or $\alpha > \bar{\alpha} > 1$, the stronger the complementarity, the lower bargaining weight a relatively more productive player gets. But when the two players have similar productivity (α near 1), i.e., $\alpha \in [\underline{\alpha}, \bar{\alpha})$, the comparative statics of $\hat{g}$ with respect to κ is not monotonic. The key channel driving this non-monotonicity is the strategic effect that is only present in the SQ game: A higher κ increases the marginal benefit to player 1 of raising x directly as well as indirectly through increasing y as well.[20] Such effect distorts, in a non-monotonic way, the counter-balancing effects of g on the surplus S represented by the two terms $S_x(dx/dg)$ and $S_y(dy/dg)$ in dS/dg. We do not expound further on the analytical details of the non-monotonic pattern of $\hat{g}$ because such pattern seems unnoticeable, as shown in Figure 4(b), the underlying effects of which are subtle to delineate without adding commensurate economic insight.

4.3 Comparison between the SM and SQ Games

Despite the non-monotonicity in the effect of κ on $\hat{g}$ near $\alpha = 1$, the comparison of the equilibrium bargaining weights in the SM and SQ games is straightforward. Figure 5 graphs both $g^*(\alpha, \kappa)$ and $\hat{g}(\alpha, \kappa)$ with respect to $\kappa \in [0, 1]$ and $\alpha \in [0, 3]$.[21]

We can see that when $\kappa = 0$, $g^*(\alpha, 0) = \hat{g}(\alpha, 0)$ regardless of $\alpha > 0$; and that when $\kappa > 0$, there is some threshold value of $\hat{\alpha}(\kappa)$ that satisfies $\partial\hat{\alpha}(\kappa)/\partial\kappa > 0$ such that if $\alpha \geq \hat{\alpha}(\kappa)$ then $\hat{g}(\alpha, \kappa) \geq g^*(\alpha, \kappa)$ and if $\alpha < \hat{\alpha}(\kappa)$ then $\hat{g}(\alpha, \kappa) < g^*(\alpha, \kappa)$.

[20]The strategic effect of κ on the equilibrium efforts is explained in Section 3.2. The non-monotonicity of $\hat{g}$ is illustrated in Figure 3 in Section 3.3 for the benchmark model with $\alpha = 1$, and in Figure 11 in Appendix A for the general model with $\alpha > 0$.

[21]Because g^* and $\hat{g}$ are defined by quadratic and quartic equations, closed-form solutions are complex to compare analytically. Figure 5 graphs the comparison for the range of $\alpha \in [0, 3]$ but the pattern extends beyond α greater than 3, as implied by Propositions 3 and 4.

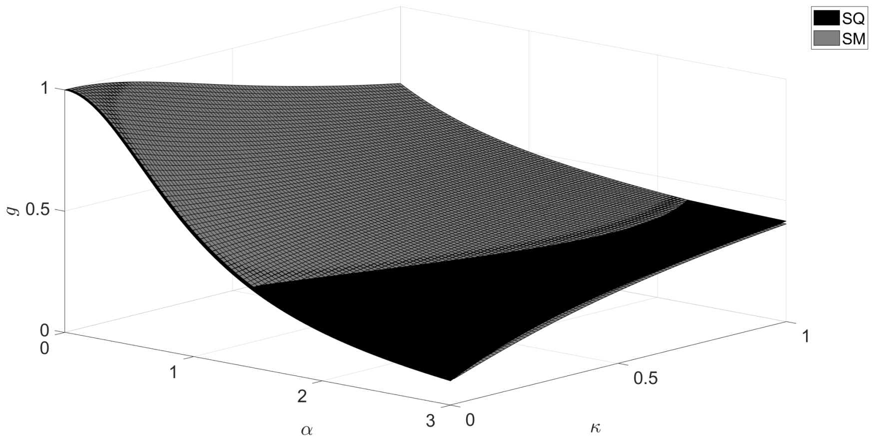
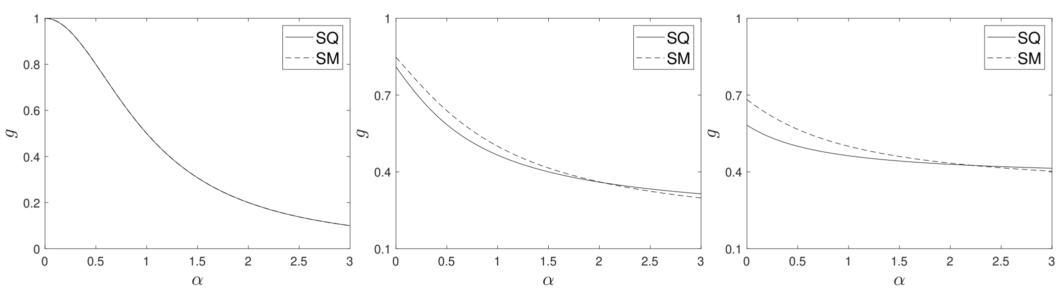


Figure 5: The equilibrium bargaining weights g^* (SM) and $\hat{g}$ (SQ)

To see the comparison more clearly, Figure 6 graphs the equilibrium bargaining weights in the SM and SQ games with respect to $\alpha \in [0, 3]$ for the three given values of $\kappa = 0$, 0.5, and 1.



(a) $\kappa = 0$ (b) $\kappa = 0.5$ (c) $\kappa = 1$

Figure 6: The equilibrium bargaining weights g^* (SM) and $\hat{g}$ (SQ), $\kappa = 0$, 0.5, 1

We summarize this comparison and the main findings of this section as follows:

Remark 2. Consider our general model given $\alpha > 0$ and $\kappa \in [0, 1]$.

(i) With no complementarity in efforts ($\kappa = 0$), the SQ game is equivalent to the SM game; in particular, $g^*(\alpha, 0) = \hat{g}(\alpha, 0)$ for all $\alpha > 0$.

(ii) The equilibrium bargaining weight for player 1 (the principal) is higher in the SQ game than in the SM game, i.e., $\hat{g}(\alpha, \kappa) > g^*(\alpha, \kappa)$, when player 2 is sufficiently more productive than player 1 given some degree of complementarity.

(iii) The equilibrium bargaining weight for player 1 increases in α given κ in both the SM and SQ games. It increases (resp. decreases) in κ if $\alpha \geq 1$ (resp. $\alpha < 1$) in the SM game, and if $\alpha \geq \bar{\alpha}$ (resp. $\alpha < \underline{\alpha}$) in the SQ game (where it is not monotonic in κ if $\alpha \in [\underline{\alpha}, \bar{\alpha})$).

Remark 2 (i) is a direct generalization of Remark 1 (i). Remark 2 (ii) implies that Remark 1 (ii) does not generally extend to situations with asymmetric productivity. When player 2 is relatively more productive, given some degree of complementarity, player 2 gets less bargaining power in the SQ game than in the SM game; on the other hand, when player 1 is relatively more productive, player 2 gets less bargaining power in the SM game than in the SQ game. The intuition can be explained using the benchmark ($\alpha = 1$), in which case (given some degree of complementarity) player 1 choosing first must be willing to give more share (than in the SM game) to player 2 to induce him to choose sequentially "enough" level of effort that further complements in maximizing the total surplus. However, when player 2 is much more productive, the effect that player 1 has on the surplus by exerting more effort is greater in the SQ game than in the SM game, which outweighs the former effect, thus weakening the intuition behind Remark 1 (ii). That is, when $\alpha > 1$ increases, both g^* and $\hat{g}$ decrease, but g^* decreases faster, implying that when player 2 becomes sufficiently more productive than player 1, player 1 choosing first does not need to forgo as much bargaining power to player 2 as in the SM game.[22]

Combining parts (i) and (ii), the managerial implication derived in the benchmark model that the principal's ability to commit matters when the partnership ex-

[22] This result relates to the logic of first-mover advantage: When player 2 is relatively more productive, player 1 being the first-mover does not have to give up as much power to player 2 as in the SM game because the more productive player 2 would have contributed to the surplus anyways.

hibits complementarity in actions continues to hold in our general model. That is, when there is no complementarity in efforts, the timing of players' effort choices does not affect the optimal bargaining weight regardless of the varying productivity of each player; however, the timing of effort choices matters in the presence of complementarity. Remark 2 (iii) describes the general comparative statics results that extend Remark 1 (iii). Aside from the non-monotonic region, the monotonic increase or decrease in κ depending on α indicates that greater complementarity means that giving more shares to the relatively less productive partner maximizes the total surplus.

5 Discussions

In this section, we offer two implications of our analysis for corporate governance and management, and discuss some issues pertaining to ownership requirements and default payoffs.

5.1 Surplus Maximization in Partnerships

The profit-sharing rule (defined by g in our model), which essentially reflects an allocation of ownership and organizational structure in a corporate governance system, governs how the revenue in the partnership is generated and distributed. In equilibrium of our model, the agent pays $w = t - y^2$ up front for his share $1 - g$ of the revenue $f(x, y)$, where g is optimally chosen to maximize the profit or surplus, $S = f(x, y) - (x^2 + y^2)$, in the partnership. After the players optimally choose their efforts, the principal collects the generated revenue $f(x, y)$ and pays $t = (1 - g)f(x, y)$ to the agent as a salary. So the principal's equilibrium payoff is $u_1 = f(x, y) - x^2 - t + w = S > 0$, and the agent's equilibrium payoff is

$u_2 = -y^2 + t - w = 0$. Thus our analysis shows that the principal essentially extracts all the surplus from the partnership in equilibrium as she is endowed with all of the bargaining power in the initial negotiation.

Because heterogeneity in productivity and the degree of complementary crucially affect the profit-sharing rule g, it is useful to examine the effects of α and κ on the surplus, and thus on the principal's equilibrium payoff. Those effects are salient and monotonic in both the SM and SQ games (see Figure 8). As shown in Figure 5, the difference in the equilibrium bargaining weights between the two games may not seem economically significant. However, the difference in the equilibrium surpluses between the two games is noticeable, as illustrated in Figure 7.

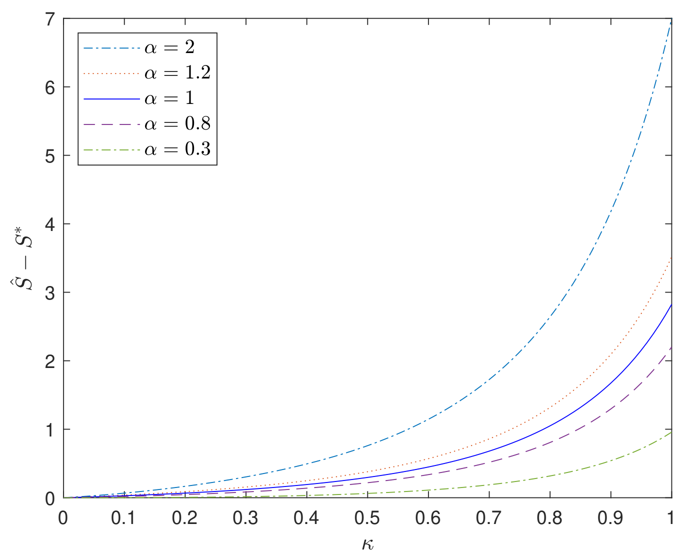


Figure 7: Difference in the equilibrium surpluses of the SQ and SM games ($\hat{S} - S^*$)

An immediate observation is that the surplus in the SQ game, $\widehat{S}$, is always higher than that in the SM game, S^*, given any values of α and κ. Also, the difference $\widehat{S} - S^*$ gets larger as either α or κ increases. In particular, the surplus gains that the principal would get when she chooses effort first relative to when the players choose simultaneously increase as κ increases in a more convex manner for a higher α. This confirms that greater complementarity leads to a higher efficacy of commitment in yielding more surplus. The timing of effort choices relates to a managerial incentive or organizational structure, and so the principal's commitment can be viewed as an institution (as part of a governance system) that affects the profit-sharing rule and thus the surplus.[23] Thus, as an implication for corporate governance, the principal can benefit more from the agent with high levels of productivity and complementarity when the principal can commit to her effort first than when not.

5.2 Substitution between Complementarity and Productivity

Continuing from the previous discussion, it is intuitive that the principal will obtain more utility from partnering with a highly productive and complementary agent than from the one who is not. We further examine the relationship between individual productivity and complementarity in shaping the principal's equilibrium payoff. To do so, we graph the combinations of α and κ values that give the same equilibrium payoff to the principal. Figure 8 shows the principal's indifference curves (or iso-utility loci) in (α, κ) space for the SM and SQ games.

We make two observations. First, the principal's indifference curves are concave to the origin in either the SM or the SQ game. This is because in equilibrium the principal shows an increasing marginal rate of substitution between the agent's productivity and complementarity.[24] Second, the marginal rate of substitution of α for κ increases differently between the SM and SQ games. In particular, the indifference curves for the SM game are steeper than those for the SQ game.

[23] This point is based on a broad definition of governance system, given by Zingales (2002), that links the way surplus is distributed and the way it is generated.

[24] That is, the equilibrium surplus is increasing and convex in both α and κ, as implied by Figure 7. We can also verify that the principal's indifference curves are never strictly convex to the origin for other values of utilities in Figure 8. These results are available from the authors upon request.

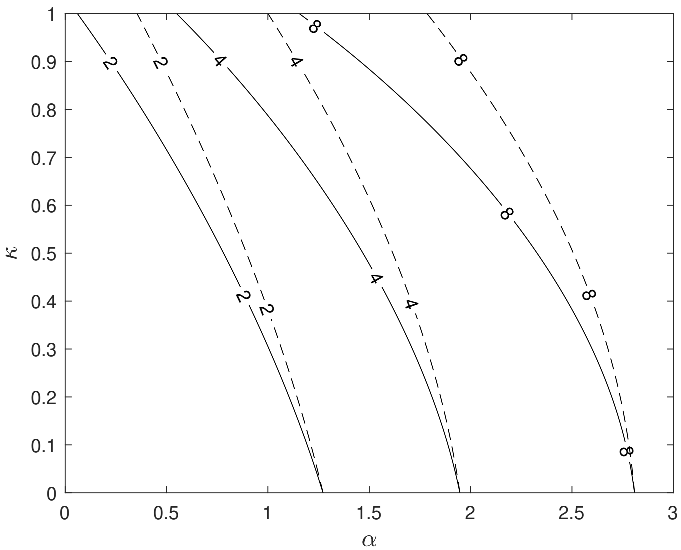


Figure 8: Principal's indifference curves for the SM (dashed) and SQ (solid) games

Those observations deliver interesting implications for management and human resources on the employee selection process and optimal hiring. Let us consider a pool of job applicants for the role of agent.[25] Those job applicants differ among the two traits: individual marginal productivity (α) and complementarity to the principal (κ). We assume that there is a distribution of traits in the labor market, and the "budget" set that the principal faces is represented by all pairs of traits (α, κ) that the applicants in the pool possess. Consider the following three potential agents in the applicant pool, denoted by circles in Figure 9. Agents P and C have strength in one trait but lack the other: Agent P is almost twice more productive than the principal but does not complement the principal ($\alpha \approx 1.94$, $\kappa = 0$), whereas Agent C strongly complements the principal but is almost half less productive than the principal ($\alpha \approx 0.55$, $\kappa = 1$). Agent A is an all-rounder with a balanced pair of traits, i.e., he is equally productive as the principal and complements her to some degree ($\alpha = 1$, $\kappa \approx 0.67$).

[25]Our model does not study the principal's problem of optimal search for a partner, i.e., endogenously choosing an agent to partner with. So we only provide a simple illustration here.

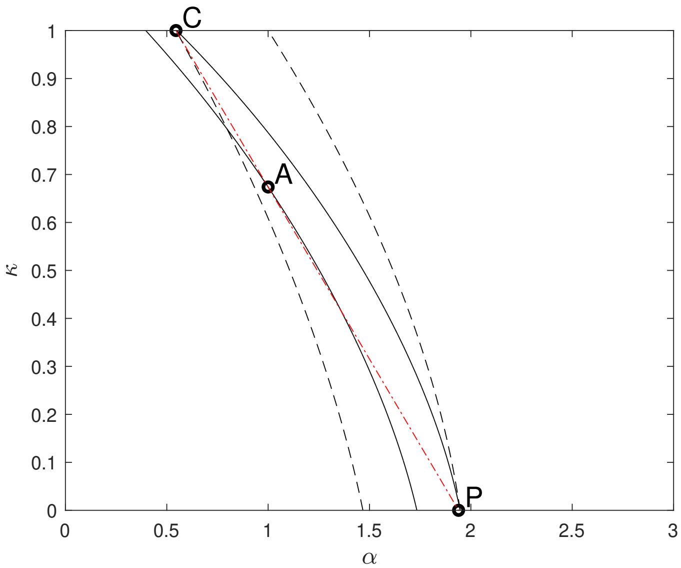


Figure 9: Principal's indifference curves and three potential agents (C, A, P).
Notes: The two solid curves are the principal's indifference curves for the SQ game. The two dashed curves are the principal's indifference curves for the SM game. The red dashed line that connects C and P represents the budget line that the principal faces in both games.

As can be seen from Figure 9, the principal's preference ordering over the three potential agents is $P \succ A \succ C$ in the SM game and $P \sim C \succ A$ in the SQ game, in terms of the equilibrium payoff that she would get from hiring each applicant as the agent for partnership. A full analysis of this problem is beyond the scope of this paper. Yet we can conclude that it is not always optimal for the principal to hire an all-rounder agent (generalist) rather than an agent with biased traits (specialist).[26]

Our second observation on the comparison of the two games indicates that complementarity is of greater value to the principal's equilibrium payoff in the SQ game than in the SM game. In other words, to give up a unit of the agent's productivity

[26]This conclusion may still be valid even when the principal's indifference curves are convex to the origin, depending on how a budget set (pool of applicants) looks like and how generalists and specialists are defined in terms of (α, κ). Here, we define a generalist as an agent who is a convex combination of the two agents at the end points of a straight budget line (e.g., C and P). Under such a definition, generalists are always better than specialists for a principal with a convex indifference curve system, but vice versa for a principal with a concave one.

and maintain the same level of utility, the principal requires a less degree of complementarity in the SQ game than in the SM game. Another way to see this is to notice that, in the example illustrated in Figure 9, the utility-maximizing points occur at P and C in the SQ game but only at P in the SM game, given the budget line (red dashed line). That is, the principal is indifferent between Agents P and C in the SQ game but she strictly prefers Agent P over Agent C in the SM game. This is because the principal's willingness to trade the agent's productivity for complementarity is lower in the SM game than in the SQ game. An implication is that there is more value to gaining complementarity and forgoing productivity of the agent in the SQ game; because the principal has an additional channel through which a higher complementarity can increase her payoff by committing to her effort first.

5.3 Constraint on Ownership

Many U.S. public firms have adopted the executive stock ownership requirements (SORs), which require that a fixed number of shares or a specified value of a stock (expressed as a multiple of salary) be held by an executive officer (Benson, Lian and Wang 2016; Brisley, Cai and Nguyen 2021; Core and Larcker 2002). In our model, the players can jointly choose any share $g \in [0,1]$ that maximizes the expected surplus in the initial negotiation. The idea of SORs can be incorporated into our model by imposing the constraint $g \geq \underline{g}$ for some $\underline{g} > 0$. This constraint may be in place for reasons other than SORs. For example, a principal may face a threat of losing ownership control if she has too low a share of her own company, or an agent may have low liquidity necessary for making an initial transfer for his share.

As shown in Figures 5 and 6, the range of the equilibrium values of g over α is smaller for a higher κ in both the SM and SQ games. In particular, the equilibrium g ranges from 0.1 to 1 when $\kappa = 0$; from 0.29 to 0.85 when $\kappa = 0.5$; and from 0.4 to

0.69 when $\kappa = 1$, over the domain from $\alpha = 3$ to $\alpha = 0$, respectively.[27] This means that the constraint $g \geq \underline{g}$, or SOR, is more likely to bind for firms with high levels of α and/or low levels of κ. Suppose we measure "firm" performance or value in our model by the surplus generated from a partnership. Then, a theoretical implication is that CEOs (whose stockholdings were at the equilibrium level) adopting a binding SOR would experience a deterioration in firm performance, broadly consistent with the empirical findings of Brisley, Cai and Nguyen (2021).[28] In this sense, a principal partnering with a highly productive but weakly complementary agent is able to extract more benefits and improve firm performance by lowering her ownership share below the executive stock ownership guideline. That is, it is not necessarily surplus-improving to adopt the SOR governance initiative, which is intended to result in better managerial incentive alignment and improved performance.

5.4 Default Payoffs

The two players receive the default payoffs of $(-x^2 + w, -y^2 - w)$ when they fail to agree on sharing the realized return of their effort choices in the final negotiation. By this time, they have already incurred their effort costs and made the transfer w. This transfer w (which is tied to the selection of g) is automatically enforced at the initial negotiation stage. In the concept of standard bargaining solution, the players immediately transfer to split the surplus in proportion to their bargaining weights. This is important for the equilibrium characterization as the players use monetary transfers to provide incentives. The equilibrium transfers are characterized by $t =$

[27]More precisely, when $\kappa = 0.5$, $g^* \in [0.29819, 0.8477]$ and $\hat{g} \in [0.31432, 0.81010]$; and when $\kappa = 1$, $g^* \in [0.40249, 0.68233]$ and $\hat{g} \in [0.41418, 0.58252]$, for the domain of α from 3 to 0.

[28]Brisley, Cai and Nguyen (2021) find that firms adopting a binding SOR underperform their non-binding SOR peers who are already in compliance with the SOR. In contrast, Benson, Lian and Wang (2016) find that firms with a binding SOR at the time of adoption have larger improvement in operating performance and better stock performance than firms with a non-binding SOR.

$(1-g)f(x,y)$ and $w = t - y^2$, which are indispensable for successful agreement in negotiation and have intuitive interpretations. The final transfer t is like a salary or dividend to the agent paid out as his share of the revenue, determined at the final negotiation; at the initial negotiation, the agent buys his share $1-g$ at the price of w, which equals his salary minus effort costs. The initial transfer w is irreversible whether or not the players fail to agree in the final negotiation.[29] Thus, both in theory and in reality, it is not plausible to consider the case in which the initial transfer w is completely nullified under disagreement in the final stage.

We may yet consider the default payoffs to be $(-x^2 + \beta w, -y^2 - \beta w)$ for some $\beta \in (0,1)$, which incorporates the idea that the agent may retrieve a fraction of the initial transfer. Such an alternative setup only complicates the analysis without changing our results or adding commensurate insight. To illustrate, consider the SM game. At the final node, by the standard bargaining solution, player 2's payoff must satisfy $-y^2 - \beta w + (1-g)f(x,y) = -y^2 - w + t$, which gives $t^* = (1-g)f(x,y) + (1-\beta)w$. At the individual decision node, the players' optimal choices of x^* and y^* are not affected given the continuation payoffs with t^*. At the initial node, by the standard bargaining solution, player 2 receives zero payoff which must be equal to $-(y^*)^2 + t^* - w$, so the optimal w must satisfy $w = t^* - (y^*)^2 = (1-g)f(x,y) + (1-\beta)w - (y^*)^2$, which yields $w^* = ((1-g)f(x,y) - y^2)/\beta$ and $t^* = ((1-g)f(x,y) - y^2 + \beta y^2)/\beta$. As can be seen here, the transfers are used to achieve the required split of the surplus.

[29]For example, when an agent buys some share of a company, he pays his share price up front; if the final negotiation fails, then he will not be receiving his portion of the realized return but is still holding the share he bought.

6 Conclusion

A successful partnership depends on partners' individual productivity, their complementarity to each other, as well as on how they make productive decisions. Those factors shape how the two partners negotiate over partnership formation and compensation schemes to share the profit from their partnership. We introduce a theoretical model that features those factors to study multi-stage negotiation process in partnerships. Our analysis identifies the effects of complementarity, productivity, and commitment on a profit-sharing rule. We offer a set of interesting implications for contracting and governance structure.

We conclude by discussing several avenues for future research. Our model does not capture the principal's problem of selecting job applicants (agents) nor the setting where multiple principals compete for agents. While we focus on the details of contractual settings that influence negotiation outcomes in partnerships, the process of selecting partners or competition may also affect ownership structure, managerial incentive schemes, productive decisions, etc. An extension in this direction is left for future work. Another avenue is to generalize our functional form of the revenue from a partnership, $f(x, y)$. While we focus on a multiplicative term κxy to capture complementarity, examining whether our results continue to hold for a more general function would be useful. Lastly, we may consider a model in which the degree of complementarity is endogenized through the principal's investment in the agent's human capital, and the investment cost is shared between the two partners. In such a situation, a hold-up problem may arise depending on how much complementarity is firm-specific.[30] Our paper offers a theoretical framework that can be extended to incorporate other related issues in teamwork and management.

[30] See Che and Sákovics (2004) and Che and Hausch (1999) for related literature.

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A Appendix: Proofs

Proof of Lemma 1. In any negotiation equilibrium, the equilibrium g must satisfy, for a given κ, the first order condition:

$$\frac{dS^*(g;\kappa)}{dg} = (2+2\kappa y^* - 2x^*)\frac{\partial x^*}{\partial g} + (2+2\kappa x^* - 2y^*)\frac{\partial y^*}{\partial g} = 0, \tag{A.1}$$

where $(2+2\kappa y^* - 2x^*) = \frac{2(1+\kappa-g-2\kappa g+\kappa g^2)}{1-\kappa^2 g(1-g)}$, $(2+2\kappa x^* - 2y^*) = \frac{2(g+\kappa g^2)}{1-\kappa^2 g(1-g)}$, $\frac{\partial x^*}{\partial g} = \frac{1-\kappa^2 g^2+\kappa-2\kappa g}{(1-\kappa^2 g(1-g))^2}$, and $\frac{\partial y^*}{\partial g} = \frac{-1+\kappa^2 g^2+\kappa-2\kappa g+\kappa^2-2\kappa^2 g}{(1-\kappa^2 g(1-g))^2}$. Canceling out and rearranging the terms, the first order condition (A.1) can be rewritten as

$$(1-2g)\Big(1+2\kappa+\kappa^2(1-g+g^2)\Big) = 0.$$

Because $1-g+g^2 \geq \frac{3}{4}$ and $\kappa^2 \geq 0$, the second term in parenthesis is always strictly positive for every $g \in [0,1]$ and every $\kappa \geq 0$; hence, the only real solution g that satisfies the first order condition is $g = 1/2$ regardless of the value of κ. Plugging in $g = 1/2$ in (3), we obtain $x^*(\kappa) = 1/(2-\kappa)$ and $y^*(\kappa) = 1/(2-\kappa)$. Recall that $t = (1-g)f(x,y)$ according to the standard bargaining solution, hence $t^*(\kappa) = x^*(\kappa) + y^*(\kappa) + \kappa x^*(\kappa)y^*(\kappa) = (4-\kappa)/(2-\kappa)^2$. Also, in the standard bargaining solution, at the initial joint decision node, player 2 must receive $d_2 + \pi_2 S^*(\kappa)$ where d_2 and π_2 are zeros and $S^*(g;\kappa)$ is obtained from (4). By setting his equilibrium payoff $u_2^* = -(y^*(\kappa))^2 + t^*(\kappa) - w$ equal to zero, the immediate transfer in equilibrium must satisfy $w^*(\kappa) = t^*(\kappa) - (y^*(\kappa))^2 = \frac{3-\kappa}{(2-\kappa)^2}$. □

Proof of Lemma 2. In any negotiation equilibrium, g in equilibrium must satisfy, for a given $\kappa \in [0,1]$, the first order condition:

$$\frac{d\widehat{S}(g;\kappa)}{dg} = (2 + 2\kappa\hat{y} - 2\hat{x})\frac{\partial \hat{x}}{\partial g} + (2 + 2\kappa\hat{x} - 2\hat{y})\frac{\partial \hat{y}}{\partial g} = 0, \tag{A.2}$$

where $(2+2\kappa\hat{y}-2\hat{x}) = \frac{2(1+\kappa-g-3\kappa g+2\kappa g^2-\kappa^2 g+\kappa^2 g^2)}{1-2\kappa^2 g(1-g)}$, $(2+2\kappa\hat{x}-2\hat{y}) = \frac{2(g+\kappa g^2)}{1-2\kappa^2 g(1-g)}$, $\frac{\partial \hat{x}}{\partial g} = \frac{1-2\kappa^2 g^2+2\kappa-4\kappa g}{(1-2\kappa^2 g(1-g))^2}$, and $\frac{\partial \hat{y}}{\partial g} = \frac{-1+2\kappa^2 g^2+\kappa-2\kappa g+2\kappa^2-4\kappa^2 g}{(1-2\kappa^2 g(1-g))^2}$. Canceling out and rearranging terms, the first order condition (A.2) is equivalent to

$$\begin{aligned} V'(g;\kappa) \equiv & -2\kappa^3(\kappa+1)g^4 + 2\kappa^2(\kappa^2-\kappa-3)g^3 + 3\kappa(2\kappa^2+4\kappa+1)g^2 \\ & - (2\kappa^3+9\kappa^2+8\kappa+2)g + (2\kappa+1)(\kappa+1) = 0. \end{aligned} \tag{A.3}$$

The function $V'(g;\kappa)$ is continuous on $g \in [0,1]$, $V'(g=0;\kappa) = (2\kappa+1)(\kappa+1) > 0$ and $V'(g=1;\kappa) = -\kappa^2 - 2\kappa - 1 < 0$ for all $\kappa \in [0,1]$. Thus the first order condition (A.3) has a solution among the $g \in (0,1)$ as a real function of κ, denoted by $\hat{g}(\kappa)$.[31] Differentiating $V'(g;\kappa)$ with respect to g and rearranging yield:

[31] A closed-form solution for $\hat{g}(\kappa)$ can be derived by using the quartic formula (Auckly 2007).

$$
\begin{aligned}
V''(g;\kappa) = &-6\kappa^3 g^2(1-\kappa) - 6\kappa(1-g) - 2\kappa(1-\kappa^2 g) - 8\kappa^4 g^3 \\
&- \Big[2 + 3\kappa^2(6g^2 - 8g + 3) + 2\kappa^3(4g^3 - 5g + 1)\Big].
\end{aligned} \tag{A.4}
$$

The terms in the bracket can be rewritten as $\Big[2(1-\kappa^3) + 3\kappa^2(6g^2 - 8g + 3)(1 - \kappa) + \kappa^3(8g^3 + 18g^2 - 34g + 13)\Big]$, where $(6g^2 - 8g + 3) = 6(g - 2/3)^2 + 1/3 > 0$, and $(8g^3 + 18g^2 - 34g + 13)$ achieves a strictly positive local minimum in the domain $g \in [0,1]$.[32] Thus $V''(g;\kappa) < 0$ for any given $\kappa \in [0,1]$ for all $g \in [0,1]$. So V'' is strictly negative whenever (A.3) holds. Hence there is a unique solution $g = \hat{g}(\kappa)$ to the first order condition (A.3). Further note that $V'(g = 1/2;\kappa) = (1/8)\kappa(\kappa+1)(\kappa-2) \leq 0$ where the equality holds iff $\kappa = 0$ and V'' is strictly negative for all $g \in [0,1]$ given $\kappa \in [0,1]$, implying that V' remains negative for $g > 1/2$. Therefore the unique solution among $g \in (0,1)$ must lie in $g \in (0, 1/2]$. That is, $\widehat{S}(g;\kappa)$ obtains its maximum value at a unique $\hat{g}(\kappa) \leq 1/2$ given $\kappa \in [0,1]$, where $\hat{g}(\kappa) = 1/2$ iff $\kappa = 0$. Plugging in $g = \hat{g}(\kappa)$ in (5), we obtain $\hat{x}(\kappa)$ and $\hat{y}(\kappa)$ in equilibrium. Using the standard bargaining solution, the equilibrium transfers are characterized by $\hat{w}(\kappa) = \hat{t}(\kappa) - (\hat{y}(\kappa))^2$ and $\hat{t}(\kappa) = (1 - \hat{g}(\kappa))f(\hat{x}(\kappa), \hat{y}(\kappa))$. $\square$

Proof of Proposition 1. Parts (i) and (ii): At the final joint decision node, the equilibrium transfer t is determined by the standard bargaining solution as in Section 3.1. In the subgame after the initial negotiation, the first order conditions to players' problems of choosing their effort levels (see (2)), become $2g(1+\kappa y) = 2x$ and $2(1-g)(\alpha + \kappa x) = 2y$. Thus we obtain

$$
x^*(g;\alpha,\kappa) = \frac{g + \alpha\kappa g(1-g)}{1 - \kappa^2 g(1-g)} \quad \text{and} \quad y^*(g;\alpha,\kappa) = \frac{\alpha(1-g) + \kappa g(1-g)}{1 - \kappa^2 g(1-g)}. \tag{A.5}
$$

[32] The local minimum is $(1/36)(1629 - 95(285)^{(1/2)}) > 0$ at $g = ((95/3)^{(1/2)} - 3)/4 \in (0,1)$.

At the initial joint decision node, the players jointly choose g to maximize the surplus $S^*(g;\alpha,\kappa) = 2(x^* + \alpha y^* + \kappa x^* y^*) - (x^*)^2 - (y^*)^2$, and w is determined by the standard bargaining solution. So g in equilibrium must satisfy, for given α and κ, the first order condition:

$$\frac{dS^*(g;\alpha,\kappa)}{dg} = (2 + 2\kappa y^* - 2x^*)\frac{\partial x^*}{\partial g} + (2\alpha + 2\kappa x^* - 2y^*)\frac{\partial y^*}{\partial g} = 0, \tag{A.6}$$

where $(2 + 2\kappa y^* - 2x^*) = \frac{2(1+\alpha\kappa-g-2\alpha\kappa g+\alpha\kappa g^2)}{1-\kappa^2 g(1-g)}$, $(2\alpha + 2\kappa x^* - 2y^*) = \frac{2(\alpha g+\kappa g^2)}{1-\kappa^2 g(1-g)}$, $\frac{\partial x^*}{\partial g} = \frac{1-\kappa^2 g^2+\alpha(\kappa-2\kappa g)}{(1-\kappa^2 g(1-g))^2}$, and $\frac{\partial y^*}{\partial g} = \frac{-\alpha+\alpha\kappa^2 g^2+\alpha\kappa^2-2\alpha\kappa^2 g+\kappa-2\kappa g}{(1-\kappa^2 g(1-g))^2}$. Canceling out and rearranging terms, the first order condition (A.6) can be rewritten as

$$\alpha^2(\kappa^2(g-1)^3 + g) + 2\alpha\kappa(2g-1) + \kappa^2 g^3 + g - 1 = 0, \tag{A.7}$$

or equivalently,

$$\begin{aligned} V'(g;\alpha,\kappa) \equiv &(1 - (1+\alpha^2)g) + 2\alpha\kappa(1-2g) \\ &+ \kappa^2(\alpha^2 - 3\alpha^2 g + 3\alpha^2 g^2 - (\alpha^2+1)g^3) = 0. \end{aligned} \tag{A.8}$$

The function $V'(g;\alpha,\kappa)$ is continuous on $g \in [0,1]$, and for all $\alpha > 0$ and $\kappa \in [0,1]$, $V'(g=0;\alpha,\kappa) = (\alpha\kappa+1)^2 > 0$ and $V'(g=1;\alpha,\kappa) = -\kappa^2 - 2\kappa\alpha - \alpha^2 < 0$.[33] Thus the first order condition (A.8) has a solution among the $g \in (0,1)$, as a real function of α and κ, denoted by $g^*(\alpha,\kappa)$. Differentiating $V'(g;\alpha,\kappa)$ with respect to g yields:

$$\begin{aligned} V''(g;\alpha,\kappa) &\equiv -(1+\alpha^2) - 4\alpha\kappa - \kappa^2(3\alpha^2 - 6\alpha^2 g + 3(\alpha^2+1)g^2) \\ &= -(1+\alpha^2) - 4\alpha\kappa - 3\kappa^2 g^2 - 3\kappa^2\alpha^2(1-g)^2. \end{aligned} \tag{A.9}$$

[33] Note that $V'(g=1;\alpha,\kappa) = 0$ iff $\alpha = \kappa = 0$, implying that the optimum occurs at $g = 1$, which is a trivial case where player 2 contributes nothing in the partnership.

We can easily see that V'' is strictly negative for all $g \in [0,1]$ given $\kappa \in [0,1]$ and $\alpha > 0$. So V'' is negative whenever $V'(g;\alpha,\kappa) = 0$. Hence there is a unique real solution $g = g^*(\alpha,\kappa) \in (0,1)$ that solves the first order condition (A.8); this condition can be rearranged as:

$$\begin{aligned} &-(\alpha^2+1)\kappa^2 g^*(\alpha,\kappa)^3 + 3\alpha^2\kappa^2 g^*(\alpha,\kappa)^2 \\ &\qquad - ((3\alpha\kappa+1)(\alpha\kappa+1)+\alpha^2)g^*(\alpha,\kappa) + (\alpha\kappa+1)^2 = 0. \end{aligned} \tag{A.10}$$

Part (iii): Evaluating $V'(g;\alpha,\kappa)$ at $g = 1/2$, we obtain $V'(g = 1/2;\alpha,\kappa) = (1/8)(\alpha+1)(\alpha-1)(\kappa^2-4)$. So $V'(g = 1/2;\alpha,\kappa) \leq 0$ iff $\alpha \geq 1$ where the equality holds iff $\alpha = 1$. This implies that the unique solution g^* to (A.10) must lie in $g \in [0,1/2)$ if $\alpha > 1$ given $\kappa \in [0,1]$, in $g \in (1/2,1]$ if $\alpha \in (0,1)$ given $\kappa \in [0,1]$, and occurs at $g = 1/2$ if $\alpha = 1$ regardless of κ. □

Proof of Proposition 2. Parts (i) and (ii): At the final joint decision node, the equilibrium transfer t is determined by the standard bargaining solution as in Section 3.2. In the subgame after the initial negotiation, from the first order conditions to players' problems of choosing their effort levels sequentially, we obtain

$$\hat{x}(g;\alpha,\kappa) = \frac{g + 2\alpha\kappa g(1-g)}{1-2\kappa^2 g(1-g)} \quad \text{and} \quad \hat{y}(g;\alpha,\kappa) = \frac{\alpha(1-g)+\kappa g(1-g)}{1-2\kappa^2 g(1-g)}. \tag{A.11}$$

At the initial joint decision node, the players jointly choose g to maximize $\widehat{S}(g;\alpha,\kappa) = 2(\hat{x}+\alpha\hat{y}+\kappa\hat{x}\hat{y}) - (\hat{x})^2 - (\hat{y})^2$, and w is determined by the standard bargaining solution. So g in equilibrium must satisfy, for given α and κ, the first order condition:

$$\frac{d\widehat{S}(g;\alpha,\kappa)}{dg} = (2+2\kappa\hat{y}-2\hat{x})\frac{\partial \hat{x}}{\partial g} + (2\alpha+2\kappa\hat{x}-2\hat{y})\frac{\partial \hat{y}}{\partial g} = 0, \tag{A.12}$$

where $(2+2\kappa\hat{y}-2\hat{x}) = \frac{2(1+\alpha\kappa-g-3\alpha\kappa g+2\alpha\kappa g^2-\kappa^2 g+\kappa^2 g^2)}{1-2\kappa^2 g(1-g)}$, $(2\alpha+2\kappa\hat{x}-2\hat{y}) =$

$\frac{2(\alpha g+\kappa g^2)}{1-2\kappa^2 g(1-g)}$, $\frac{\partial \hat{x}}{\partial g} = \frac{1-2\kappa^2 g^2+\alpha(2\kappa-4\kappa g)}{(1-2\kappa^2 g(1-g))^2}$, and $\frac{\partial \hat{y}}{\partial g} = \frac{-\alpha+2\alpha\kappa^2 g^2+\kappa-2\kappa g+2\alpha\kappa^2-4\alpha\kappa^2 g}{(1-2\kappa^2 g(1-g))^2}$. Canceling out and rearranging terms, the first order condition (A.12) is equivalent to

$$\begin{aligned} V'(g;\alpha,\kappa) &\equiv -2\kappa^3(\kappa+\alpha)g^4 + 2\kappa^2(\kappa^2-\alpha\kappa-3\alpha^2)g^3 \\ &+ 3\alpha\kappa(2\kappa^2+4\alpha\kappa+1)g^2 - (2\alpha\kappa^3+(8\alpha^2+1)\kappa^2+8\alpha\kappa+\alpha^2+1)g \\ &+ (2\alpha\kappa+1)(\alpha\kappa+1) = 0. \end{aligned} \tag{A.13}$$

The function $V'(g;\alpha,\kappa)$ is continuous on $g \in [0,1]$, and for all $\alpha > 0$ and $\kappa \in [0,1]$, $V'(g=0;\alpha,\kappa) = (2\alpha\kappa+1)(\alpha\kappa+1) > 0$ and $V'(g=1;\alpha,\kappa) = -\kappa^2-2\alpha\kappa-\alpha^2 < 0$. Thus the first order condition (A.13) has a solution among $g \in (0,1)$, as a real function of α and κ, denoted by $\hat{g}(\alpha,\kappa)$. Differentiating $V'(g;\alpha,\kappa)$ with respect to g yields:

$$\begin{aligned} V''(g;\alpha,\kappa) &= -8\kappa^3(\kappa+\alpha)g^3 + 6\kappa^2(\kappa^2-\alpha\kappa-3\alpha^2)g^2 \\ &+ 6\alpha\kappa(2\kappa^2+4\alpha\kappa+1)g - (2\alpha\kappa^3+(8\alpha^2+1)\kappa^2+8\alpha\kappa+\alpha^2+1). \end{aligned} \tag{A.14}$$

When (A.13) holds, taking into account that a solution is among $g \in (0,1)$, the condition $V''(g;\alpha,\kappa) < 0$ can be equivalently written as

$$\begin{aligned} &(-12g^3+12g^2-2)\alpha^2\kappa^2 + \big((-6g^4-4g^3+6g^2)\kappa^2+3(g^2-1)\big)\alpha\kappa \\ &+ (-6\kappa^4 g^4+4\kappa^4 g^3-1) < 0. \end{aligned}$$

The term $(-12g^3+12g^2-2) < 0$ in the domain $g \in [0,1]$, achieving its (local) maximum of $-2/9$; the term $(-6g^4-4g^3+6g^2) < 1$ in the domain $g \in [0,1]$, achieving its (local) maximum of $5/8$, so the term $((-6g^4-4g^3+6g^2)\kappa^2+3(g^2-1)) < 0$ for $g \in [0,1]$ given $\kappa \in [0,1]$; and the term $(-6\kappa^4 g^4+4\kappa^4 g^3) < 1$ for any given $\kappa \in [0,1]$ as it achieves its (global) maximum of $\kappa^4/8 \leq 1/8$, so the term $(-6\kappa^4 g^4+4\kappa^4 g^3-1) < 0$ given $\kappa \in [0,1]$. Therefore, the above inequality holds; that is, $V''(g;\alpha,\kappa) < 0$ for any given $\alpha > 0$ and $\kappa \in [0,1]$ for all $g \in [0,1]$ whenever (A.13) holds. Hence there is a unique solution $g = \hat{g}(\alpha,\kappa)$ to the first order condition (A.13). Part (iii): Evaluating $V'(g;\alpha,\kappa)$ at $g = 1/2$, we obtain $V'(g = 1/2;\alpha,\kappa) = (1/8)(\kappa^2 - 2)(2(\alpha^2 - 1) + \alpha\kappa + \kappa^2)$. So if $\alpha \geq 1$, then $V'(g = 1/2;\alpha,\kappa) \leq 0$ always where the equality holds iff $\alpha = 1$ and $\kappa = 0$. But when $\alpha < 1$, $(2(\alpha^2 - 1) + \alpha\kappa + \kappa^2) \geq 0$ if and only if $\alpha \geq 1/2$ and $(1/2)(\sqrt{8 - 7\alpha^2} - \alpha) \leq \kappa \leq 1$ where the equality holds iff $1/2 \leq \alpha < 1$ and $\kappa = (1/2)(\sqrt{8 - 7\alpha^2} - \alpha)$. These imply that the unique solution $\hat{g}$ to (A.13) must lie in $g \in [0, 1/2]$ iff $\alpha \geq 1$, or $1/2 \leq \alpha < 1$ and $(1/2)(\sqrt{8 - 7\alpha^2} - \alpha) \leq \kappa \leq 1$; it occurs at $\hat{g}(\alpha,\kappa) = 1/2$ iff $1/2 \leq \alpha \leq 1$ and $\kappa = (1/2)(\sqrt{8 - 7\alpha^2} - \alpha)$. $\square$

Proof of Proposition 3. Part (i): By the implicit function theorem, the implicit derivative $\frac{dg^*(\alpha,\kappa)}{d\alpha}$ can be obtained from (A.8) evaluated at the optimum g^*:

$$\frac{dg^*(\alpha,\kappa)}{d\alpha} = \frac{-2\big(\alpha\kappa^2(g^* - 1)^3 + \alpha g^* + \kappa(2g^* - 1)\big)}{\alpha^2(3k^2(g^* - 1)^2 + 1) + 4\alpha\kappa + 3\kappa^2(g^*)^2 + 1}. \tag{A.15}$$

The denominator in (A.15) is strictly positive. From Proposition 1, $g^* > 1/2$ when $\alpha \in (0,1)$ and $g^* \leq 1/2$ when $\alpha \geq 1$. Suppose that $\alpha \in (0,1)$. Then in the numerator of (A.15), $\alpha\kappa^2(g^* - 1)^3 + \alpha g^* > 0$ because $\kappa \in [0,1]$ and $|g^* - 1| < g^*$, and $\kappa(2g^* - 1) \geq 0$; so the numerator in (A.15) is strictly negative. Now suppose that $\alpha \geq 1$. Taking into account that g^* satisfies (A.8), the terms $\big(\alpha\kappa^2(g^* - 1)^3 + \alpha g^* + \kappa(2g^* - 1)\big)$ in the numerator can be rewritten as

$$\begin{aligned}
&(1/\alpha)\big(\alpha^2(\kappa^2(g^* - 1)^3 + g^*) + \alpha\kappa(2g^* - 1)\big) \\
&= (1/\alpha)\big(-\alpha\kappa(2g^* - 1) + (1/2 - g^*) + (1/2 - \kappa^2(g^*)^3)\big) > 0,
\end{aligned}$$

where the equality holds using (A.7). The positive sign follows because $g^* \leq 1/2$; so the numerator in (A.15) is strictly negative. Thus $\frac{dg^*(\alpha,\kappa)}{d\alpha} < 0$ for any given

$\kappa \in [0, 1]$. Part (ii): By the implicit function theorem, the implicit derivative $\frac{dg^*(\alpha,\kappa)}{d\kappa}$ can be obtained from (A.8) evaluated at the optimum g^*:

$$\frac{dg^*(\alpha,\kappa)}{d\kappa} = \frac{-2(\alpha^2\kappa(g^*-1)^3 + \alpha(2g^*-1) + \kappa(g^*)^3)}{\alpha^2(3\kappa^2(g^*-1)^2+1) + 4\alpha\kappa + 3\kappa^2(g^*)^2 + 1}. \tag{A.16}$$

The denominator in (A.16) is strictly positive. By Proposition 1, $g^* > 1/2$ when $\alpha \in (0, 1)$ and $g^* \leq 1/2$ when $\alpha \geq 1$. Suppose that $\alpha \in (0, 1)$. Then in the numerator of (A.16), $\alpha^2\kappa(g^*-1)^3+\kappa(g^*)^3 \geq 0$ because $\kappa \in [0, 1]$ and $|g^*-1| < g^*$, and $\alpha(2g^*-1) > 0$; so the numerator in (A.16) is strictly negative when $\alpha \in (0, 1)$. Now suppose that $\alpha \geq 1$. Then $\alpha^2\kappa(g^*-1)^3 + \kappa(g^*)^3 \leq 0$ because $\alpha \geq 1$ and $g^* \leq 1/2$, and $\alpha(2g^*-1) \leq 0$. So the numerator in (A.16) is strictly positive when $\alpha > 1$ and is zero when $\alpha = 1$. □

Graphical Illustration of Proposition 4. Part (i): Figure 10 graphs the cross-partial $\frac{\partial^2 S}{\partial \hat{g}\partial\alpha}$ in the (α, κ)-space for $\alpha \in (0, 3]$ and $\kappa \in [0, 1]$. The pattern of graph extends beyond $\alpha > 3$. We observe that $\frac{\partial^2 S}{\partial \hat{g}\partial\alpha} < 0$ for any given $\kappa \in [0, 1]$ and $\alpha > 0$. By Topkis' Monotonicity Theorem, $\hat{g}$ strictly decreases in $\alpha > 0$.

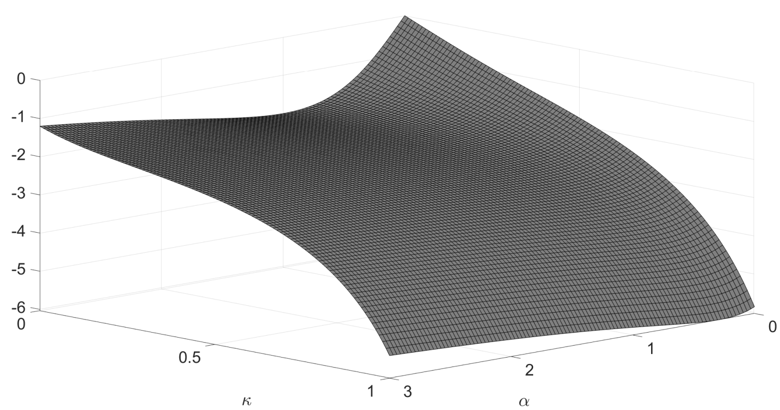

Figure 10: The cross partial $\frac{\partial^2 S}{\partial \hat{g}\partial\alpha}$ in the SQ game

Part (ii): In Figure 4(b), it is quite clear to see that $\hat{g}$ monotonically decreases in κ for some small values of α, say $\alpha < 0.8$, and increases in κ for some larger values of α, say $\alpha > 1.2$. But the pattern of $\hat{g}$ with respect to κ for some intermediate values of α is not clearly observed. Figure 11 shows graphically the $\hat{g}$ with respect to κ for different values of α.

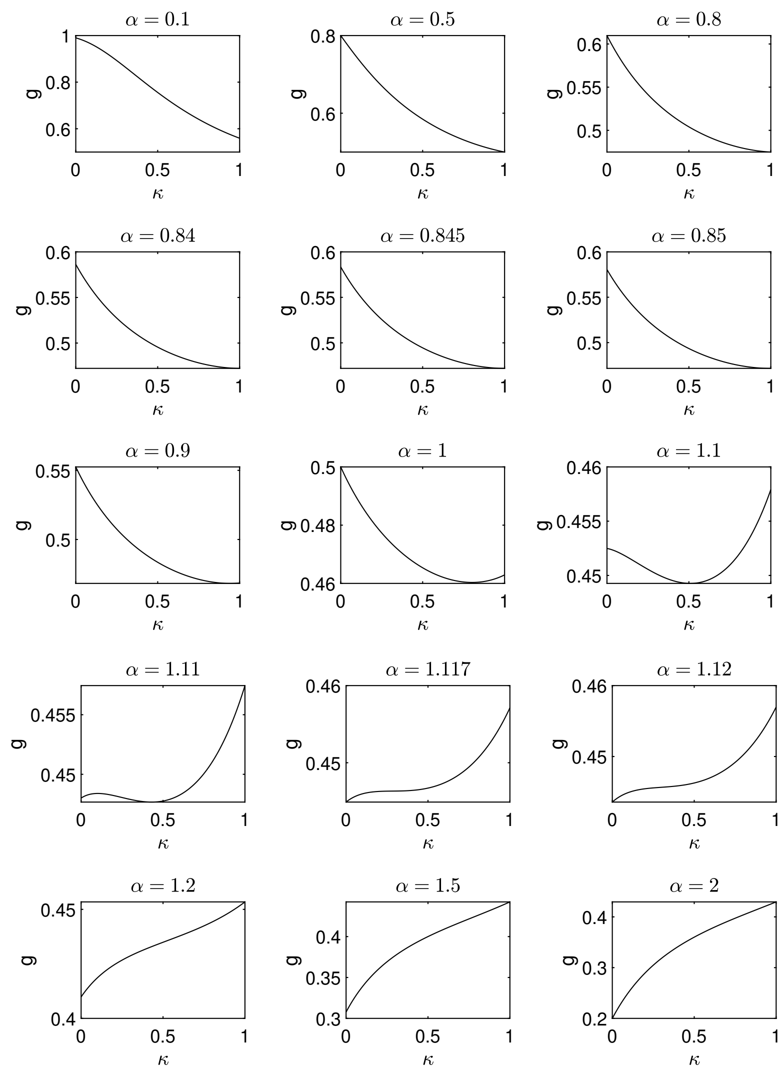

Figure 11: The $\hat{g}$ with respect to κ for different α

While the monotonic decrease or increase of $\hat{g}$ in κ is observed for $\alpha < 0.845$ and $\alpha > 1.117$, the $\hat{g}$ exhibits non-monotonic patterns for α near $\alpha = 1$. $\square$